

IPC00-0270

Implementation of Tuned Vibration Absorbers for Above Ground Pipeline Vibration Control

Mark A. Norris

Keith R. Ptak

Ben A. Zamora

Lord Corporation, Cary, NC 27511

James D. Hart

SSD, Inc, Reno, NV 89509

ABSTRACT

An overview of recent developments of tuned vibration absorbers (TVAs) for vibration suppression is presented. The paper summarizes some popular theory for analysis and optimal tuning of these devices, discusses various design configurations, and reviews the recent application of TVAs to control wind-induced oscillations of pipelines above the Arctic Circle. Although the wind-induced pipeline vibrations are relatively small, the accumulation of vibration cycles can cause fatigue at pipeline joints. The TVAs used in this application have reduced the RMS displacements of the pipeline by as much as a factor of seven. Additionally, the paper introduces a new overhead TVA installation on the pipeline for accommodating environmental considerations.

INTRODUCTION

Since its invention almost a century ago [Frahm, 1911], the tuned vibration absorber (TVA) has been an important engineering tool for vibration suppression. This simple device, often consisting of a reaction mass and a spring element with appropriate damping, has proven very effective for reducing severe vibrations of machinery, buildings, bridges and many other mechanical systems with relatively low cost. It is by and large an engineering design challenge to make effective TVAs under constraints of weight, damping and physical dimensions.

Over the years, many TVA design configurations have been developed. Different optimal tuning rules have also been studied for tonal and broadband applications. In the meantime, the utility of TVAs has progressed beyond isolating machinery at the frequency of a rotating unbalance. Considerable attention has been given to problems of controlling the modal and forced response of complex continuous systems. Such systems include civil engineering structures including above ground pipelines.

A recent application of TVAs is to control wind-induced oscillations of pipelines above the Arctic Circle. Vibrations of the pipeline result from cyclic lift forces associated with the vortex shedding phenomenon. Depending upon the wind conditions, as many as ten vibration modes of a pipeline span can be excited. Although the amplitudes of vibration are relatively small, the accumulation of vibration cycles can cause fatigue at the pipeline joints.

This paper presents a brief overview of the applications of TVAs, including those used for pipelines. In section 2, we discuss some fundamental aspects of TVAs including an impedance coupling method for performance estimation, several optimal tuning rules and a summary of design variations of these devices. In section 3, we discuss some contemporary applications of TVAs including above ground pipelines. Note that in the literature, TVAs are also referred to as tuned mass dampers, dynamic vibration absorbers, and auxiliary mass dampers. In this paper, we use the term TVA.

TVA FUNDAMENTALS

This section begins with a discussion of the impedance coupling method and continues with the optimal tuning rules for TVAs and with a summary of design variations of these devices.

A common design approach for TVAs in vibration control applications comprises an analysis of the full system including TVAs [Snowdon, 1968, and Den Hartog, 1956]. Theoretical work on the application of TVAs to simple one and two dimensional structures can be found in many sources [Sun, et. al., 1995]. Such an analysis leads to an estimate of the vibration control performance provided by the TVAs and insight into various tuning strategies. In practice, however, a complete mathematical model of a complex system under consideration is often not readily available.

Another analysis and design approach favored by engineers, known as the impedance coupling method, is often used [Ewins, 1984]. This method does not need the complete model of the structure, as it only requires the driving point impedance at the TVA attachment points and the impedance of the TVAs. The impedance of the TVA is generally chosen to be sufficiently large in the frequency range of interest so that the structural response at the attachment point is adequately reduced by the TVA.

Consider an arbitrary primary structure and a TVA as a second substructure. The coordinates to describe these substructures are x_p and x_t , and the coordinate used to describe the coupled system is x_c (Figure 1). The substructures may both possess several degrees of freedom, even though only a few coordinates are included for the purpose of coupling the components together. When the system is described in terms of the coupling degrees-of-freedom, the system is not fully described in-space. However, these coordinates will exhibit the full range of resonances possessed by the coupled system.

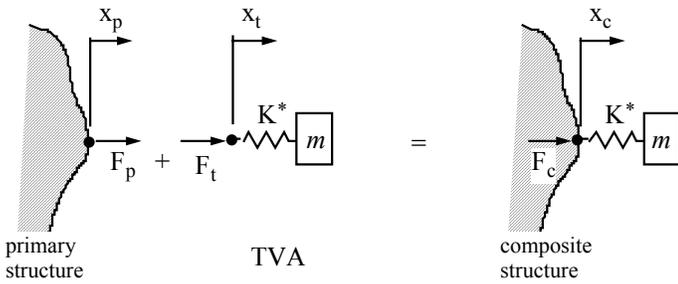


Figure 1. Impedance coupling of a TVA to a primary structure in the direction of the defined coordinates.

When a harmonic force F_p is applied to the primary structure as shown in Fig. 1(a), in steady-state, we obtain an equation in the frequency domain given by

$$V_p = F_p / Z_p, \quad (1)$$

where V_p and Z_p denote the velocity and the driving point impedance at the attachment point, respectively. Note that in steady-state, the velocity V_p and the displacement x_p are related by $V_p = i\omega x_p$, where $i = \sqrt{-1}$. Similarly, the equation for the TVA is given by

$$V_t = F_t / Z_t. \quad (2)$$

By imposing the conditions of continuity and equilibrium to the coupled system, we have the following

$$V_c = V_p = V_t, \quad \text{and} \quad F_c = F_p + F_t. \quad (3)$$

Hence, using Eqs. (1) to (3), we have the input-output relation for the coupled system:

$$V_c = F_c / Z_c, \quad \text{and} \quad Z_c = Z_p + Z_t, \quad (4a, b)$$

where Z_c and V_c are the driving point impedance and velocity of the coupled system at the TVA attachment point. Note that the input-output relationship of the coupled system is determined in terms of the properties of the independent substructures. Eq. (4b) suggests that in order to obtain a 20 dB reduction in the structural response at a given frequency, for example, the coupled system impedance must be ten times greater than the impedance of the primary system at that frequency. This requirement gives one constraint on the impedance of the TVA. This constraint together with other design requirements (e.g., maximum allowable TVA mass and stroke) will determine all the TVA parameters.

The impedance Z_p of the primary structure may be modeled or measured directly using, for example, an instrumented impact hammer and an accelerometer. Likewise, the TVA impedance may also be measured directly, or it can be computed from the transmissibility – a quantity relatively easy to measure. By definition, the transmissibility is given by

$$T = x_o / x_t, \quad (5)$$

where x_t is the base displacement and x_o is the TVA mass displacement. A force F_t applied to the TVA base is related to the acceleration of the TVA mass m by

$$F_t = m\ddot{x} \quad (6)$$

Also, the constitutive relation between the force F_t and the TVA deflection is given by

$$F_t = K^*(x_t - x_o), \quad (7)$$

where K^* is the complex dynamic stiffness of the resilient element of the TVA. From the above equations, we obtain the transmissibility of the TVA as

$$T = K^* / (K^* - m\omega^2). \quad (8)$$

By combining Eqs. (5) and (6), we obtain

$$F_t = -m\omega^2 T x_t, \quad (9)$$

from which the impedance of the TVA at its attachment point is derived as

$$Z_t = mi\omega T. \quad (10)$$

An Example of TVA Design for Tonal Applications

Consider a primary structure that, at low frequencies, responds in a predominantly spring-like fashion with stiffness K_S at a point where a TVA is to be placed. Suppose that this structure is excited by a tonal disturbance F at a frequency ω_0 within this low frequency regime. Furthermore, suppose that reliability, displacement, and/or operating environment

considerations require that the TVA mass displacement is sufficiently low. To accomplish this, the design engineer must incorporate damping into the resilient element. We wish to find the required mass of the TVA in order to reduce vibration in the primary structure by a factor of α .

Suppose that the TVA contains an elastomeric resilient element with hysteretic damping. The complex stiffness of the resilient element has the form

$$K^* = K(1 + i\delta), \quad (11)$$

where K is the real part of the stiffness and δ is the damping or loss factor [Ewins, 1984]. In order to obtain a factor of α reduction in the structural response at the excitation frequency ω_0 , the coupled system impedance Z_c , given by Eq. (4b), must be α times greater than the impedance of the primary system at ω_0 . In other words, after dividing Eq. (4b) by Z_p , we have

$$\left| \frac{Z_c}{Z_p} \right| = \left| 1 + \frac{Z_t}{Z_p} \right| = \alpha, \quad \text{at } \omega_0 \quad (12)$$

By setting the TVA undamped natural frequency to the excitation frequency, $K/m = \omega_0^2$, Eqs. (8), (10) and (11) can be combined to show that the impedance of the TVA at ω_0 becomes

$$Z_t = m\omega_0 \left(\frac{1}{\delta} + i \right). \quad (13)$$

Eq. (13) suggests that when the damping factor δ is much less than unity, the primary structure sees a primarily dissipative impedance of the TVA at $\omega = \omega_0$ because it has a large real part. Interestingly, this dissipative impedance component at $\omega = \omega_0$ is inversely proportional to the TVA damping (the lower the TVA damping, the greater the effective TVA dissipative effect).

Since the primary system is spring-like, its impedance is $Z_p = -iK_s/\omega_0$. Eq. (12) can be rewritten as

$$\left| 1 - \frac{m\omega_0^2}{K_s} + i \frac{m\omega_0^2}{K_s} \frac{1}{\delta} \right| = \alpha, \quad \text{at } \omega_0 \quad (14)$$

or

$$\left(1 - \frac{m\omega_0^2}{K_s} \right)^2 + \left(\frac{m\omega_0^2}{K_s} \right)^2 \left(\frac{1}{\delta^2} \right) = \alpha^2 \quad (15)$$

When $\alpha \gg 1$ and $\delta \ll 1$, Eq. (15) becomes

$$\frac{m\omega_0^2}{K_s} \frac{1}{\delta} \cong \alpha, \quad (16)$$

which yields

$$m \cong \alpha \delta \frac{K_s}{\omega_0^2}. \quad (17)$$

Eq. (17) suggests that the required TVA mass is directly proportional to the attenuation desired and is also proportional to the damping factor of the TVA. Hence, as the damping diminishes, the required mass approaches zero, when the TVA remains tuned to the excitation frequency. Under this circumstance the TVA mass displacement approaches infinity. In particular, it can be shown that the displacement is inversely proportional to the damping factor:

$$\left| \frac{x_o - x_t}{x_t} \right| = \frac{1}{\delta}, \quad \text{at } \omega_0 \quad (18)$$

which is derived directly from the TVA transmissibility in Eq. (8). A finite level of damping must be incorporated into the TVA to provide a suitable tradeoff between TVA mass and displacement.

It should be pointed out that the conclusions drawn from this simple example hold qualitatively for more complex structures.

Design of TVAs for Broadband Vibration

In tonal applications, TVAs are generally tuned at the disturbance frequency. At its resonance, the TVA drives the structure to impede structural motion. Furthermore, the TVA alters the frequency response of the primary system and introduces another resonance in the composite system. For tonal applications, low TVA damping results in improved performance for a given mass. In practice, however, damping is incorporated in order to maintain a reasonable tradeoff between the TVA mass and its displacement. Hence, the design effort for this class of applications is focused on having precise tuning of the TVA to the disturbance frequency and controlling damping to an appropriate level.

In broadband vibration control problems, TVAs are generally designed to add damping to and change the resonant characteristics of a primary structure in order to maximally dissipate vibrational energy over a range of frequencies. Therefore, the TVA design rules applicable to tonal applications are not appropriate to broadband applications. When designed appropriately, TVAs can perform particularly well when the primary structure is lightly-damped. Figure 2 shows an example where a single damped TVA beneficially changes the dynamic characteristics of a lightly damped structure near the tuning frequency and provides additional damping at other frequencies. Although the TVA adds another resonance to the system, the resonant responses are well damped as compared to those of the original system.

A classical design method for broadband applications, known as the equal-peak method, is studied in [Snowdon, 1968, and Den Hartog, 1956]. Figure 3 shows the transmissibility of a lightly damped single degree of freedom oscillator (primary system) coupled with a lightly damped TVA tuned to the resonance of the oscillator. Referring to this figure, the equal-peak method [Snowdon, 1968] is stated as: *a TVA "is said to be most favorably tuned and damped when the*

two maximum values of the transmissibility that lie outside the frequency range (ω_a and ω_b) are equal, because this common value will be exceeded if the absorber is tuned differently. The classical procedure is to assume that the maximum values of the transmissibility actually occur at the frequencies ω_a and ω_b .”

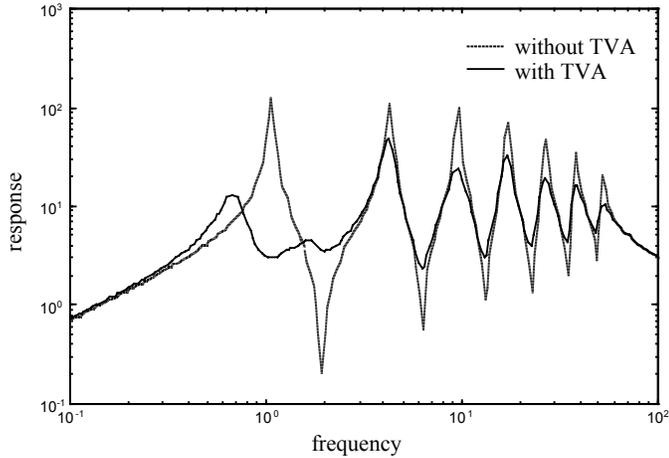


Figure 2. The effect of a TVA on the broadband response of a continuous structure. The TVA mass is 5% of the structural mass and is tuned to the first mode of the structure. The structure has 3% damping and the TVA has 20% damping.

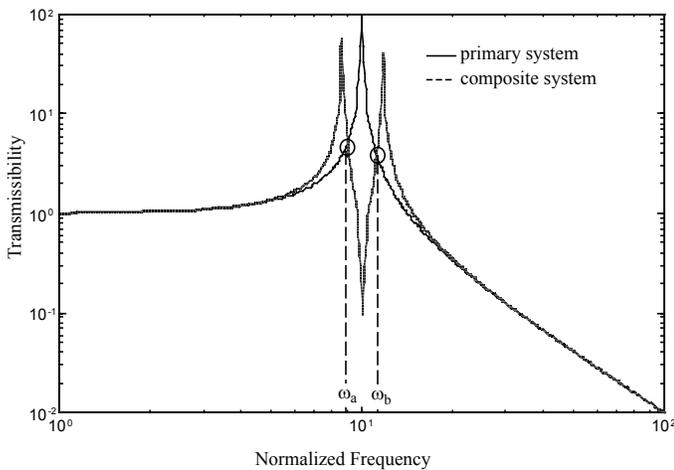


Figure 3. Transmissibility curves of a single degree-of-freedom primary system and the coupled system with a TVA tuned to the resonance of the primary system. Encircled intersections are invariant to TVA damping.

By following this method, a "favorable" tuned frequency ω_n of the TVA can be derived as [Snowdon, 1968]

$$\omega_n = \frac{1}{1+\mu} \omega_0, \quad (19)$$

where ω_0 is the resonance frequency of the primary single degree-of-freedom system and μ is the mass ratio of the TVA mass m to the primary system mass M given by

$$\mu = m / M. \quad (20)$$

If the dynamic stiffness of the TVA resilient element is of the viscous type given by

$$K^* = K (1 + i 2 \omega / \omega_n \zeta), \quad (21)$$

where ζ is the viscous damping ratio; a "favorable" damping ratio ω_0 can also be obtained based on this method [Snowdon, 1968]

$$\omega_0 = (3\mu / [8(1 + \mu)])^{1/2}. \quad (22)$$

From Eq. (19), we see that the tuning frequency ω_n is always less than the primary system resonance ω_0 . Figure 4 shows the equal-peak method for TVA design applied to the single degree of freedom oscillator. It can be seen from the figure that the performance improves as the TVA mass increases. This trend is the same as in tonal applications.

Besides the equal-peak method, there have been many studies of other optimal designs of TVAs for broadband applications [Sun, et. al., 1995].

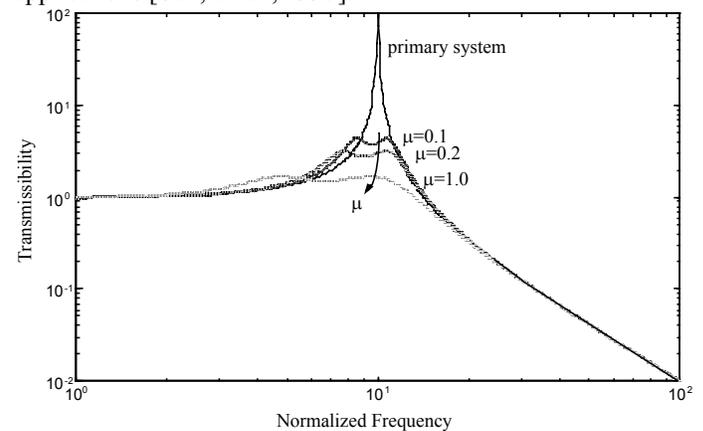


Figure 4. Transmissibility curves of a single degree-of-freedom primary system and the coupled system with a TVA designed with the equal-peak method. Three mass ratios are shown for the coupled system.

Design Variations of TVAs

Conceptually, a TVA consists of a reaction mass and a resilient element. The simplest design is a spring-mass oscillator. While, this simple design remains a favorable choice [Hart, J. D., et. al., 1992], over the years, there have been many design variations of TVAs. Pendulum-type absorbers have been developed in [Fujinami, T., et. al., 1991]. A rotary TVA similar to a pendulum-type absorber (except that the restoring force is provided by a centrifugal force rather than the gravity) has been developed in [Paul, W. F., 1969]. A mechanical rotary TVA is studied in [Duh, J. and Miao, W., 1983], while a magnetic dynamic absorber using eddy currents to provide damping is developed in [Kobayashi, H. and Aida,

S., 1993]. There are also studies of multi-degree-of-freedom TVAs [Duh, J. and Miao, W., 1983]. By allowing the reaction mass to have more than one degree of freedom, it is possible to develop a single-mass TVA that can reduce vibration at several frequencies and thereby achieve weight savings.

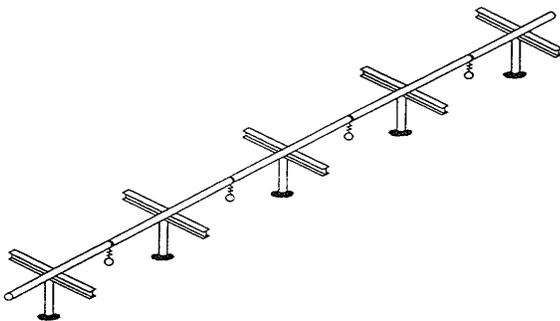
Creating novel TVAs for new applications using different mechanisms and physical principles, while maintaining constraints of weight, size and cost, remains an interesting design challenge, and will surely continue to attract attention from researchers and engineers.

Recent Applications of Passive TVAs

Applications of TVAs to civil engineering structures include vibration suppression of tall buildings [Hrovat, D., et. al., 1983], bridges [Siwiecki, K. J., 1972], offshore platforms [Grindmeiar, B. L., et. al., 1989], and pipelines [Hart, J. D., et. al., 1992]. In these applications, the excitation is often broadband, e.g., earthquake, wind loading and sea waves. The broadband excitation may excite low order modes characterized by large motions in the structure and can be particularly damaging. TVAs are generally tuned near or at frequencies associated with these low order modes and can also increase the overall damping in the structure.

A recent application of TVAs is to control wind-induced oscillations of pipelines above the Arctic circle. Vibrations of the pipeline result from cyclic lift forces associated with the vortex shedding phenomenon. Depending upon the wind conditions, as many as ten vibration modes of a pipeline span can be excited. For a 12 to 18 meter span, the first ten modes are all below 5 Hz. Although the amplitudes of vibration are relatively small, the accumulation of vibration cycles can cause fatigue at pipeline joints [Hart, J. D., et. al., 1992]. Arctic conditions make TVA design particularly challenging – maintaining TVA stiffness and damping properties over a broad low temperature range lies at the heart of this challenge. One TVA per span has been implemented as shown in Figure 5. Due to the high modal density of the pipeline span below 5 Hz, a TVA tuned in this frequency range will always be near or at a resonance of the pipeline within the variations of the TVA stiffness induced by environmental changes. Motion of the pipeline excites the TVA so that it dissipates vibrational energy.

Figure 5. Arctic pipeline with one TVA per span [Hart, J. D., et. al., 1992].



Early TVA configurations on Arctic Pipelines consisted of masses suspended by elastomeric springs dangling below the pipeline. The TVAs used in this application have reduced the

RMS displacements of the pipeline by as much as a factor of seven [Hart, J. D., et. al., 1992]. The TVA is constructed from a 23 kg to 34 kg weight supported by a series of elastomeric components in shear deformation. This configuration is shown in Figure 6.

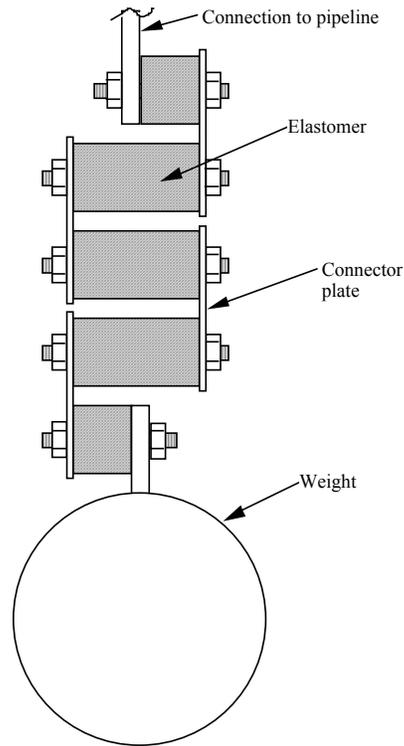


Figure 6. The TVA configuration for the arctic pipeline (early configuration; from Hart, et. al., 1993).

Environmental concerns have recently required the TVAs to be attached to the pipeline on the top side of the pipeline. Indeed, below pipeline suspended TVAs could potentially interfere with wildlife and could be a safety hazard for snowmobilers. Due to these reasons, a top-side TVA configuration was developed.

Figure 7 illustrates a schematic of an above pipeline TVA. Although the design appears relatively straightforward, many configurations were designed and tested. The challenge of the design lies in developing an elastomeric system that is able to perform reliably under severe environmental conditions. The configuration implemented in Figure 7 (also see installation picture in Figure 8) has two elastomeric discs that work in shear and support the TVA mass. The elastomeric discs in effect work as a rotational spring. Note that masses can be easily added or removed from the TVA for tuning the device. Since the impedance of the above pipeline configuration is nearly identical to that of the below pipeline configuration, these TVAs reduce the RMS displacements of the pipeline by as much as a factor of seven as well.

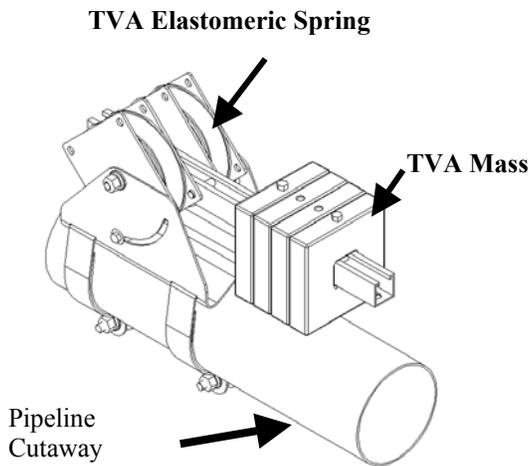


Figure 7. The TVA configuration for the arctic pipeline (above pipeline configuration).



Figure 8. Picture of the Installed TVA on the arctic pipeline (above pipeline configuration).

Overview of Above Ground Pipeline Configurations

Typical arctic cross country pipeline configurations consist of a longitudinal thermal anchor followed by 10 to 40 (or more) straight run spans with uniform span lengths (ranging 40 to 65 feet), an expansion loop, then another 10 to 40 (or more) straight run spans and another anchor. The most important feature of this structural configuration is long (up to say 1000 feet) straight pipe runs of up to 40 (or more) spans. Analytical and experimental investigations of these systems have identified useful patterns related to the vibration modes in the vertical plane of the pipeline, which are the modes most excited by vortex shedding. The modal trends indicate that the behavior of multi-span systems is essentially bounded by the behavior of a single span system. For example in a 20 span run, the vertical modes will occur in “bands” of 20 modes. For a 10 span run, the vertical modes will occur in bands of 10 modes. Each band of modes is distributed over a range of frequencies that are bounded by the frequencies of a single span with pinned and fixed end conditions, respectively. The bounding frequencies of the modes with pinned and fixed end

conditions are closely approximated by the closed form equations for the frequencies of a single span beam. Depending on the Reynolds number of the wind flow around the pipe, any of these modes can be excited by vortex-shedding.

For a 15 span run, the 15 modes in the first (“primary” mode) band have shapes where there is essentially one lobe per span. These primary modes are the most important modes for wind-induced vibration analysis. The shape of first of these 15 modes will have pinned-pinned boundary conditions within each span with adjacent spans undergoing motion in the opposite direction (i.e., odd spans moving up while even spans are moving down). The shape of the last of these 15 modes will have fixed-fixed boundary conditions within each span and all spans undergoing motion in the same direction. The frequency of the first of the primary modes is referred to as ω_{pp} (pp for pinned-pinned boundary conditions within a single span) while the frequency of the last of the primary modes is referred to as ω_{ff} (ff for fixed-fixed boundary conditions within a single span). For the lower frequency modes (near ω_{pp}) within this group, the maximum bending stress is at a midspan location, as would be the case in a pinned-end beam under uniform load. For the higher frequency modes (near ω_{ff}) within this group, the maximum bending stress is at a support location, as would be the case in a fixed-end beam under uniform load. Based on the closed form solutions for a single span, it can be shown that the primary frequency with fixed end conditions (at the high frequency end of the first band) is about 2.3 times the primary frequency with pinned end conditions (at the low frequency end of the first band).

Based on field test data, it has been determined that typical above ground pipeline systems are very lightly damped with modal damping ratios less than 0.5% of critical. As is in the case of a lightly-damped TVA, it can be shown that the amplitude of the steady-state response of the pipeline under resonant wind-induced vibration conditions is inversely proportional to the damping ratio (see Eq. (18)). Hence, doubling the pipeline damping ratio can reduce the resonant response amplitude by a factor of 2. Because the pipeline damping levels are so low, it does not take an excessive amount of additional energy dissipation to provide a substantial relative increase in the system damping.

Application to Above Ground Pipeline Configurations

The concept of the TVA systems is that resonant motion of the structure will tend to excite resonant or near-resonant motions of the added TVAs. For TVAs with visco-elastic characteristics, relative motion in the TVAs will result in the dissipation of energy in excess of that which was dissipated in the structure without the added dampers.

For conventional primary mode mitigation, a family of three “primary” TVAs, each tuned to a separate target frequency, is used to provide substantial added damping over the frequency band containing the primary vertical mode shapes which are subject to wind-induced vibration. The first, second, and third TVAs in the family are referred to as the “L”, “M” and “H” TVAs, for devices with “low”, “medium” and “high” target frequencies, respectively. The “L” TVA is tuned to the lowest frequency in the band, the “H” TVA is tuned to

the highest frequency in the band, and the "M" TVA is tuned to a frequency approximately midway between the L and H TVA frequencies. The triple tuning is distributed throughout the multi-span pipeline system by suspending the individual "L", "M", and "H" TVAs (one per span) in a repeated, alternating pattern (H-L-M-H-L-M, etc.) from span to span.

Examples of Successful Applications

TVAs have been installed on the Alaskan North Slope for over 10 years. A total of over 60 different pipeline configurations with diameters ranging from 2 inches to 24 inches and spans ranging from 30 feet to 115 feet. Approximately 30,000 spans on the North Slope have been effectively mitigated.

Effectiveness of this TVA Mitigation

The performance of TVAs has been documented in detailed field tests. The general observations are as follows:

- TVAs tend to induce a more broad-banded (multi-modal) response than that observed in the bare pipeline.
- The triple tuning concept can provide effective mitigation over the range of frequencies for cross-county arctic pipeline configurations.
- The TVAs provide WIV amplitude reduction measures ranging from 1.5 to about 7 (pipeline vibration amplitudes were reduced under all wind conditions).
- The performance of the TVA devices is not influenced by wind direction or the source of excitation (e.g., vortex induced excitation or wake buffeting excitation).
- The effectiveness of the TVAs increase essentially linearly with increasing amplitude in the bare pipeline (i.e., the larger the amplitude in the bare pipeline, the better the TVA performance).

Advantages of TVAs

- TVAs are very easy to install.
- TVAs have been proved effective under arctic conditions.
- Field measured pipeline WIV amplitude reduction factors of up to 7.
- Method mitigates WIV vibration regardless of the source of the wind loading (e.g., vortex shedding or buffeting)

SUMMARY

Tuned vibration absorbers have been an effective engineering tool for vibration reduction for nearly a century. They provide vibration control solutions for tonal and broadband applications with relatively low cost. Furthermore, these devices represent classic design challenges. In this paper, we have presented a brief review of the recent application of these devices for above ground pipelines for reducing the onset of wind induced structural fatigue.

REFERENCES

Den Hartog, J. P., 1956, *Mechanical vibration*, McGraw Hill, New York.

Duh, J. and Miao, W., 1983, "Development of Monofilar Rotor Hub Vibration Absorber," Contractor Report (Contract NAS1-16700), 166088, NASA.

Ewins, D. J., 1984, *Modal Testing: Theory and Practice*, Research Studies Press.

Frahm, H., 1911, "Device for Damping Vibrations of Bodies," US Patent 989,958.

Fujinami, T., Yamamoto, S. and Sone, A., 1991, "Dynamic absorber using lever and pendulum mechanism for vibration control of structure (The method of deciding parameters for the system)," *Transactions of the Japan Society of Mechanical Engineers, Part C*, Vol. 57, pp. 3490-3496.

Grindmeiar, B. L., Campbell, R. B. and Wesselink, B. D., 1989, "A Solution for Wind-Induced Vortex-Shedding Vibration of the Harmony and Heritage Platforms During Transpacific Tow," *Proceedings of the 31st Annual Offshore Technology Conference*.

Hart, J. D., Sause, R., Ford, G. W. and Row, D. G., 1992, "Mitigation of Wind-Induced Vibration of Arctic Pipeline Systems," *Proceedings of the 11th International Conference on Offshore Mechanics and Arctic Engineering*.

Hrovat, D., Barak, P. and Rabins, M., 1983, "Semi-active versus passive or active tuned mass dampers for structural control," *Journal of Engineering Mechanics*, Vol. 109, pp. 691-705.

Kobayashi, H. and Aida, S., 1993, "Development of a Houde damper using magnetic damping," *Vibration Isolation, Acoustics, and Damping in Mechanical Systems, ASME Design Engineering Division*, Vol. 62, pp. 25-29.

Paul, W. F., 1969, "Development and Evaluation of the Main Rotor Bifilar Absorber," *Proceedings of 25th American Helicopter Society Forum*.

Snowdon, J. C., 1968, *Vibration and Shock in Damped Mechanical Systems*, John Wiley & Sons, New York.

Sun, J. Q., Jolly, M. R., and Norris, M. A., 1995, "Passive, Adaptive and Active Tuned Vibration Absorbers – A Survey", *Transactions of the ASME*, Vol. 117, pp. 234-242.