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A MATERIAL MODEL FOR PIPELINE STEELS

by

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ABSTRACT

Experience has shown that the pipe steel used in the Trans-Alaska Pipeline System has complex properties that must be taken into account in making safety assessments of the pipe. To obtain a better understanding of the steel behavior, a detailed test program has recently been undertaken. The test results have been used to develop a nonlinear model of the steel for use in stress and deformation analysis of the pipeline. This paper first outlines the model, and shows that it captures important aspects of the steel behavior, including progressive yielding and anisotropy. The paper then shows how the values of the model parameters can be calculated from experimental stress-strain data, and how the model can be used for the analysis of pressurized pipelines, accounting for interaction between hoop and longitudinal stress. The theory is based on von Mises yield and the Mroz plasticity model.

1. INTRODUCTION

An important aspect of the design and operation of the Trans Alaska Pipeline System (TAPS) is the curvature imposed on the pipeline by differential settlement of the surrounding soil. Settled areas can be identified reliably using the NOWSCO GEOPIG [1] "smart pig" which measures the in-situ geometry of the pipeline. Changes in the pig orientation as it passes through the lines are used to compute the pipeline curvature.

SSD has developed a curvature screening tool that can be used to help make decisions on whether or not excavation and repair or releveling of the pipeline is necessary. The screening tool, which is based in part on SSD's PIPLIN [2] computer program, estimates the pipeline moment curvature relationships accounting for internal pressure and axial force effects and inelastic behavior of the pipe steel. The tool has been calibrated against the "Berkeley Tests" [3] which were conducted on full-scale TAPS pipe joints during the 1970's. One of the most important parameters influencing the serviceability decisions is the stress-strain behavior of the pipe material. The TAPS pipe material is known to exhibit significant anisotropic properties, which must be taken into account to obtain accurate predictions of the experimental wrinkling behavior [4].

As part of the screening tool development effort, a detailed material testing program has been undertaken by Alyeska at Southwest Research Institute (SwRI) in San Antonio, Texas. Based on the results from these tests, SSD has developed an 8 parameter model that captures the essential features of the observed inelastic steel behavior. The 8 parameter model is used together with the Mroz plasticity theory [5] to model the steel, assuming that the pipe wall is in a state of biaxial stress. This paper presents a description of this material model. The steps in the presentation are as follows.

(1) In Section 2, the behavior observed in recent tests of TAPS pipe steels is described.
(2) In Section 3 a method is presented for converting the experimental stress-strain curves into a convenient mathematical form.
(3) The method presented in Section 3 considers uniaxial behavior of the steel, under axial stress or hoop stress alone. In Section 4 the Mroz theory for behavior under biaxial stress (combined axial and hoop) is reviewed.
(4) The Mroz theory requires that the stress-strain curve be expressed in a "multilinear" form, consisting of a number of straight lines. In Section 5 a method is described for converting the mathematical functions into multilinear form.

2. BEHAVIOR OF TAPS STEELS

2.1 Shape of Stress-Strain Curve

Stress-strain curves for pipeline steels are usually obtained experimentally using tension test specimens [6,7]. Figure 2.1 shows the overall form of the engineering stress-strain curve for a tension specimen cut from TAPS pipe joints. This curve has five main regions, as follows.

(1) A linear region in which the specimen is elastic, with an elastic modulus near 30000 ksi (206843 MPa). At the end of this region (the proportional limit) the specimen begins to yield.
(2) A curvilinear region in which the specimen yields and strain hardens. The tangent modulus in this region becomes progressively smaller.
(3) An essentially linear region in which the specimen continues to strain harden, with a tangent modulus that is nearly constant. This modulus is small, with a typical value about 1% of the elastic modulus.
(4) A long region in which the strength is roughly constant.
(5) A strain softening region in which the strength reduces and the specimen ultimately fractures. This region begins at strains of about 12%, and the specimen typically fractures at strains above 20%.

For analysis of pipelines up to incipient wrinkling, the first three of these regions are the most important, since the strains at the onset of wrinkling are below 5%. For analysis, therefore, the stress-strain behavior to be considered is as shown in Figure 2.2. This curve is divided into three regions, namely a linear elastic region, a curved "transition" region, and an essentially linear "fully plastic" region. Note that the term "fully plastic" is not strictly correct, since the steel still has a finite hardening modulus.

2.2 Anisotropy

Figure 2.3 shows tension stress-strain curves for longitudinal and hoop specimens cut from a typical 48 inch (122 cm) diameter, 0.462 inch (1.17 cm) and 0.562 inch (1.43 cm) thick joint (length) of X65 TAPS pipe. The two curves are substantially different, with a larger proportional limit stress and a shorter curvilinear region in the hoop direction than in the longitudinal direction. The hoop curve has an essentially horizontal region up to about 1% strain, after which it has essentially the same strain hardening modulus as the longitudinal curve.

The steel thus behaves as an anisotropic material, with different behavior in the longitudinal and hoop directions. The reason for the anisotropy appears to be related to the manufacturing process, when the pipe is cold expanded in the hoop direction to satisfy tolerances on roundness and diameter. Comparative analyses that account for or ignore the anisotropy show that it has a substantial effect on the analysis results, and hence that it should be accounted for [4].

2.3 Specified Minimum Yield Strength

For TAPS, steels of X60, X65 and X70 grades are used, with specified minimum yield stress (SMYS) values of 60, 65 and 70 ksi (414, 448, and 483 MPa), respectively. According to specification API 5L [6], the yield stress value is defined as the stress at 0.5% tensile strain in the hoop, not the longitudinal, direction.

To satisfy API-5L, the actual hoop stress at 0.5% tensile strain (YS_H) must equal or exceed the SMYS. All of the recently tested joints of TAPS pipe have YS_H values that exceed the SMYS values with a ratio YS_H/SMYS ranging from 1.03 to 1.15. Because of the anisotropy, the actual longitudinal stress at 0.5% tensile strain (YS_L) may be less than SMYS. For the typical X65 steel shown in Figure 2.3, YS_H=71.8 ksi (495 MPa) and YS_L=65.2 ksi (450 MPa).

2.4 Aspects to be Captured in a Model

From the preceding discussion, the following aspects of behavior should be considered when modeling pipeline steels.

(1) Linear elastic behavior up to the proportional limit.
(2) Yielding with a progressively decreasing tangent modulus.
(3) A final linear region with a small strain hardening modulus. It is typically not necessary to consider strains larger than about 3%.
(4) Anisotropy, giving different stress-strain curves in the longitudinal and hoop directions.

In the next sections a steel model that accounts for these aspects is described for uniaxial and biaxial behavior.

3. UNIAXIAL MODEL

3.1 Parameters for Longitudinal Relationship

The shape of the longitudinal stress-strain relationship for the model is first defined using six parameters, as shown in Figure 3.1. These six parameters are as follows.

(1) The initial elastic modulus, E_{start}
(2) The final strain hardening modulus, E_{end}, for strains larger than about 1%.
(3) The nominal yield strength, σ_{YL}. This is the point where the elastic and strain hardening slopes intersect (note that $\varepsilon_{YL} = \sigma_{YL}/E_{start}$).
(4) A factor, α_L defining the proportional limit stress.
(5) The strain at which the "fully plastic" state is reached, defined by a factor β_L.
(6) A parameter, γ, that defines the shape of the relationship between the proportional limit (at a stress equal to $(1-\alpha_L)\sigma_{YL}$) and the "fully plastic" state (at a strain equal to $(1+\beta_L)\,\varepsilon_{YL}$).

3.2 Additional Parameters for Anisotropy

If the pipe steel behaved isotropically, the stress-strain relationship would be the same in the longitudinal and hoop directions. As noted, however, the steel behaves anisotropically, with different relationships. Two additional parameters are used to define the amount of anisotropy, as shown in Figure 3.2. These parameters are as follows.

(7) The stress difference between the hoop and longitudinal tension curves at large strains, DSY.
(8) A factor, α_H, defining the hoop tension proportional limit stress.

As explained later, the complete hoop tension stress-strain relationship is defined using these two parameters plus the six parameters for the longitudinal tension stress-strain relationship.

3.3 Choosing Parameter Values

If values are given for the above 8 parameters, the longitudinal and hoop stress-strain relationships can be constructed. Typically, however, the stress-strain relationships will be known from test results, and parameter values must be chosen to match these known relationships.

Given the relationships for the longitudinal and hoop directions, both defined as a series of stress-strain points, the steps are as follows. The sequence is illustrated in Figure 3.3.

(1) Choose a series of experimental points that define the longitudinal relationship.
(2) Choose the value of E_{start} to match the initial elastic slope. Also choose the proportional limit stress.
(3) Choose the value of E_{end} to match the strain hardening slope for strains larger than about 1%.
(4) Hence get the values of σ_{YL} and α_L.
(5) Choose the longitudinal strain at the start of the "fully plastic" region. Hence get the value of β_L.

(6) Set up a pair of skew axes covering the curved segment of the relationship. Map the stress-strain points in this segment from X' and Y' to a pair of normalized orthogonal axes, X and Y, as shown in Figure 3.3.
(7) Assume an ellipsoidal curve in the normalized space, with the form $X^{\gamma}+Y^{\gamma}=1$. Using least squares minimization, find γ that gives the best fit with the data. A value $\gamma=1$ is a straight line, and $\gamma=2$ is a circle.
(8) Map the curve back to the stress-strain axes (X' and Y'). Confirm that a good fit has been obtained to the experimental points.

The six parameters required to compute the longitudinal tension relationship are now known. For the hoop relationship the additional steps are as follows (see Figure 3.2).

(9) Choose a series of experimental points that define the hoop stress-strain relationship.
(10) Use the same values of E_{start} and E_{end} as for the longitudinal relationship.
(11) Choose the difference in strength, DSY, at about 2% strain. Hence get the value of σ_{YH}.
(12) Choose the hoop proportional limit stress. Hence get the value of α_H.

The two additional parameters required to compute the hoop tension relationship are now also known.

3.4 Multilinear Form for Analysis

The 8-parameter model defines stress-strain relationships that are curved in the "transition" region. As shown in Section 4, for analysis using the Mroz theory these curves must be approximated by multilinear curves made up of straight segments. The procedure for setting up the multilinear curves is described in the Section 5.

3.5 Comparison: Analytical and Experimental Curves

The procedure for selecting the 8 parameters has been applied to the Alyeska/SwRI detailed test results referred to earlier, and the values of the parameters for each tested steel have been established. These steel models are given names starting with "DT", for "detailed test". Test results are also available for the steels from the full scale "Berkeley Test" specimens [3]. The values of the 8 parameters for these steels have also been established. These steel models are given names starting with "B", for "Berkeley".

For strains up to 3%, Figures 3.4 through 3.6 compare the analytical tension stress-strain curves for steels "DT1", "DT3" and "DT6" with the experimental data from which the curves were derived (from Joints 1, 3 and 6 of the test series). The analyses were performed using PIPLIN, and hence are for the multilinear forms of the 8-parameter model.

For these steels the agreement is very close for longitudinal tension, which is to be expected since it is the primary basis of the 8-parameter model. The agreement is also close for hoop tension, except for the part of the curve just after first yield, where the experimental data shows an essentially zero slope.

For strains up to 5%, Figure 3.7 compares the tension test data for Berkeley Test pipe joint number 1007 with the analytical steel "B60-B". The experimental hoop tension curve is based on ring expansion test data. This curve indicates that the hoop strength is substantially larger than the longitudinal strength at a strain of about 2%, corresponding to a DSY value of about 2.5 ksi. This is different from the behavior measured in the recent detailed tests, which showed essentially the same longitudinal and hoop tension strengths at 2% strain, or a zero DSY value. It is possible, therefore, that the difference between the longitudinal strength and the hoop strength is a consequence of the ring expansion test method.

For strains up to 5%, Figure 3.8 compares the tension test data for Berkeley Test pipe joint number 3743 with the analytical steel "B65-A" with a zero DSY value. A longitudinal tension stress-strain curve for this joint is given in [3], but no hoop tension curves are available. The hoop tension curve was developed by assuming that the ratio of hoop to longitudinal tension strength at a strain of 0.5% was similar to that for joint 1007.

For strains up to 5%, Figure 3.9 compares the tension test data for Berkeley Test pipe joint number K1212, which is a spiral welded joint, with the analytical steel "B60S-H". A comparison of the longitudinal and hoop tension curves shows that they are nearly identical, indicating that the material is essentially isotropic. This is reasonable, since spiral welded pipe is not cold expanded during manufacture. Figure 3.9 shows that the analytical and experimental results agree closely up to a strain of about 1.5%, and that the actual steel then strain hardens and becomes significantly stronger than the analytical steel.

4. BIAXIAL BEHAVIOR

4.1 Von Mises Yield: Effect of Hoop Tension

For analysis using PIPLIN, the pipe is modeled essentially as a beam, and the beam moment-curvature behavior is calculated by considering the stress-strain behavior of longitudinal "fibers" of the pipe cross section. If the only significant stresses on these fibers were longitudinal stresses, as is usually the case in a beam, the fiber stress-strain relationship would be the longitudinal strain relationship for the pipe steel. However, a pipe also has internal pressure loading, which causes hoop stresses. Because of the internal pressure, a fiber is in a state of biaxial stress, with combined hoop and longitudinal stresses. As a result, the longitudinal fibers, have effective stress-strain relationships that are different from the basic longitudinal relationship. This is a consequence of the von Mises theory for yield of steel under multi-axial stresses.

The basic effect is illustrated in Figure 4.1. Figure 4.1(a) shows a simple elastic-perfectly-plastic stress-strain relationship (i.e., a relationship with no strain hardening). Figure 4.1(b) shows the corresponding von Mises ellipse for yield in a biaxial state of stress. If the stress point lies within the ellipse the steel is elastic; if the stress point lies on the ellipse the steel is yielded. Stress points outside the ellipse are not allowed for an elastic-perfectly-plastic stress-strain relationship. As shown in Figure 4.1(b), if the hoop stress is zero, the longitudinal yield strengths are equal to the uniaxial values, and are also equal in tension and compression. As shown in Figure 4.1(c), however, as the internal pressure, and hence the hoop stress, is increased, the longitudinal yield strengths change, and become different in tension and compression. In particular, the longitudinal yield strength in compression can be substantially smaller than the uniaxial yield strength.

It may be noted that the hoop and longitudinal stresses are not the only stresses in a pipe. They are, however, the dominant stresses in most cases. It is assumed that the other stresses, which include shear and radial stresses, are small and can be ignored.

4.2 Mroz Theory: Strain Hardening Behavior

Figure 4.2 shows a steel that has a multi-linear stress-strain curve, similar to the curve for an actual pipe steel. The material still yields according to the von Mises theory, but now its behavior is defined not by a single von Mises ellipse but by a series of ellipses. If the stress point is inside the smallest ellipse the steel is elastic, and its modulus is the elastic modulus. Each time the stress point reaches a new ellipse the modulus changes to a smaller value. An elastic-perfectly-plastic steel is a special case, with a zero strain hardening modulus after the first ellipse. As shown in Figure 4.2, if the steel is subjected to uniaxial stress, in either the hoop or longitudinal direction, the stress-strain curve is the uniaxial curve.

The Mroz theory specifies how the ellipses move as the steel yields, and hence models the behavior of the steel under biaxial stress (the Mroz theory applies to full multi-axial stress conditions, but only the special case of biaxial stress is assumed for the pipe wall). The theory postulates that the ellipses translate without changing size or shape, which is the well-known kinematic hardening assumption. The theory also specifies the direction of movement for each ellipse. Essentially, any ellipse moves so that when the stress point reaches the next larger ellipse, the yielding ellipses do not overlap. If the steel unloads, and returns to an elastic state, any ellipses that have moved remain in their shifted positions until the steel re-yields.

Figure 4.3 shows how the ellipses for the steel in Figure 4.2 would move for a stress path that involves (1) application of hoop tension (due to internal pressure), (2) addition of longitudinal compression (due to axial force and bending moment), causing the steel to yield, and (3) removal of the hoop tension (depressurization), with elastic unloading. In this example, hoop stress is first applied (Path 0-1-2). First yield occurs at Point 1, when the stress point reaches the first (smallest) von Mises ellipse. As the steel yields and strain hardens, the Mroz theory predicts that the first ellipse moves as shown. At Point 2 axial stress is added along Path 2-3-4. As before the first ellipse moves. At Point 3 the stress point reaches the second ellipse. Both the first and second ellipses then move. Finally the hoop stress is removed. The steel unloads elastically along Path 4-5, with no further movement of the ellipses. The ellipses thus remain in their shifted positions. One consequence of this is that if the stresses were reversed, along Path 5-4-3-2-1-0, the behavior would be elastic, whereas along the original Path 0-1-2-3-4-5 the steel yielded.

This example illustrates the essential features of the Mroz theory. The theory considers general loading paths, including non-radial loading (e.g., apply hoop tension then add longitudinal tension or compression) and cyclic loading (e.g., cyclic yielding in tension and compression).

4.3 Anisotropy

As noted previously, tests on hoop and longitudinal specimens cut from TAPS pipe show that the pipe steel has significantly different behavior in the hoop and longitudinal directions (i.e., the steel is anisotropic). In the basic Mroz theory the behavior is the same in both directions (i.e., the steel is isotropic), as shown in Figure 4.2. As noted, the cause of the anisotropy appears to be cold working of the steel during the manufacturing process, when the pipe is expanded to satisfy tolerances on roundness and diameter. This effect is predicted qualitatively by the Mroz theory, and can be incorporated into the theory as a simple extension.

The effect of hoop expansion is shown in Figure 4.4. If the steel is yielded in the hoop direction (by cold expansion) and then unloaded, the von Mises ellipses are shifted as shown. Hence, for subsequent loading the stress-strain curves in the hoop and axial directions are different. The curves are also different in tension and compression. Hence, the steel behaves anisotropically.

One way to include anisotropy in the Mroz theory is to "prestrain" the steel by yielding it in the hoop direction, exactly as in Figure 4.4. This approach has been used in a number of pipe wrinkling studies [4]. An alternative approach is to achieve the same effect by specifying initial shifts for the von Mises ellipses. In Figure 4.4, if it is specified that the centers of the two inner ellipses are not initially at the origin, but are shifted along the hoop axis, the effect is the same as imposing hoop prestrain. The initial shift method is more flexible than the prestraining method, and for more recent studies it has replaced prestraining.

5. CONVERSION OF 8-PARAMETER MODEL PARAMETERS TO INPUT REQUIRED FOR MROZ THEORY

5.1 Multilinear Model with Shifts

The 8-parameter model described in Section 3 defines curvilinear stress-strain relationships for longitudinal and hoop tension (see Figures 3.1 and 3.2). For analyses using the Mroz theory, this relationship must first be approximated by multilinear curves, using a number of straight segments. These multilinear curves must then be converted to corresponding von Mises ellipses, plus a strain hardening modulus for each ellipse. As shown in Figure 5.1, the size of each ellipse is defined by a uniaxial yield stress, σ_Y, and its initial location (to account for hoop prestraining) is defined by a hoop shift, S_H. The procedure for calculating the multilinear curves, and hence the von Mises ellipses, is described in this section.

5.2 Longitudinal Tension Curve

Figure 5.2 shows stress-strain curves for longitudinal and hoop tension. The differences between the curves have been exaggerated for clarity. Each multilinear curve is defined by points O, A, B, and C, plus points T1, T2, T3, etc. in the "transition" region. The number of points in the transition region can be specified, and is usually larger than 3. The stress-strain coordinates of the points on the longitudinal tension curve are calculated as follows (see Figure 5.2).

(1) Point O is the origin.
(2) Point A is calculated using E_{start}, σ_{YL} and α_L.
(3) Point C is calculated using E_{end}, σ_{YL} and a specified maximum strain (the strain at Point C).
(4) Point B is calculated using E_{start}, E_{end}, σ_{YL} and β_L.
(5) Axes X'-Y' and X-Y are set up for the transition region. Points T1, T2, etc. are spaced at equal angles in the X-Y space. The X-Y coordinates of these points are calculated, then transformed to X'-Y' coordinates, and hence to stress-strain coordinates.

5.3 Hoop Tension Curve

The stress-strain coordinates of the points on the hoop tension curve are calculated as follows (see Figure 5.2).

(1) Point O is the origin.
(2) The stress σ_{YH} is calculated using σ_{YL} and DSY.
(3) Point A is calculated using E_{start}, σ_{YH} and α_H.
(4) Point C is calculated using E_{end}, σ_{YH} and the specified maximum strain (at Point C).
(5) Point B is calculated such that the hoop curve in the transition region has the same proportions as the longitudinal curve. This ensures that the tangent moduli for the linear segments are the same for both curves, which is a requirement of the Mroz theory. This is why the parameter β_H is not one of the eight parameters. In effect, the value of β_H is chosen so that the transition regions have the same proportions for both curves.
(6) As before, axes X'-Y' and X-Y are set up for the transition region, points T1, T2, etc. are spaced at equal angles in the X-Y space, and the X-Y coordinates of these points are calculated. These coordinates are the same for both curves. As before, these coordinates are transformed to X'-Y' coordinates, and hence to stress-strain coordinates.

5.4 Ellipse Sizes and Shifts

The longitudinal and hoop curves are now defined by corresponding pairs of points (O, A, T1, T2, ..., B, C). A von Mises ellipse must be set up for each pair of points. The sizes and shifts for the ellipses must be such that the correct longitudinal and hoop tension curves are obtained if the resulting steel model is analyzed for uniaxial stresses. The longitudinal and hoop compression curves then follow from the Mroz theory.

Figure 5.1 shows a typical ellipse. Given a pair of longitudinal and hoop stresses, (σ_L, σ_H), for a pair of corresponding points on the two stress-strain curves (e.g., Points A_L and A_H shown in Figure 5.2), the size, σ_Y, and hoop shift, S_H, of the ellipse are calculated by solving the following two simultaneous equations.

(1) To get the correct hoop stress :

$$\sigma_H - S_H = \sigma_Y$$

(2) To get the correct longitudinal stress :

$$\sigma^2_L - S^2_H + \sigma_L S_H = \sigma^2_Y$$

As the earlier examples show (Figures 3.4 through 3.9), the theoretical and experimental curves are in close agreement.

6. CONCLUSION

Tests show that TAPS pipeline steels have complex behavior, with curvilinear stress-strain relationships and a substantial amount of anisotropy. In order to develop rational methods for the analysis of pipes, particularly for estimating pipe curvature capacities, it is important for the analysis models to capture the important aspects of the steel behavior.

This paper has described an "8-parameter" steel model for mathematical representation of the behavior of TAPS pipeline steels. This model captures most aspects of the steel behavior, as observed in tests on a number of pipe specimens. This paper has shown how the parameter values for the model can be calculated from experimental stress-strain data, and how the model can be used in stress and deformation analyses using the PIPLIN computer program. This paper has also presented a brief explanation of the Mroz plasticity theory that is used to account for interaction between hoop and longitudinal stresses.

7. ACKNOWLEDGMENT

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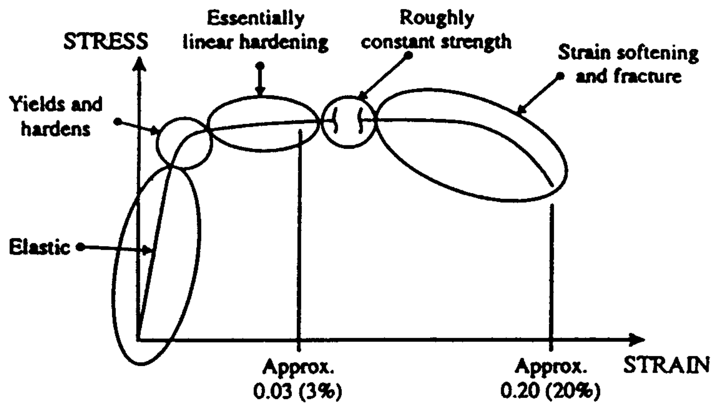


FIGURE 2.1 SHAPE OF STRESS-STRAIN CURVE FOR TAPS PIPE STEEL

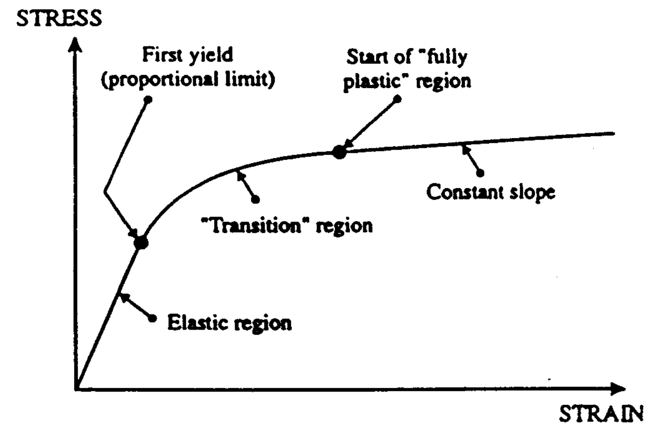
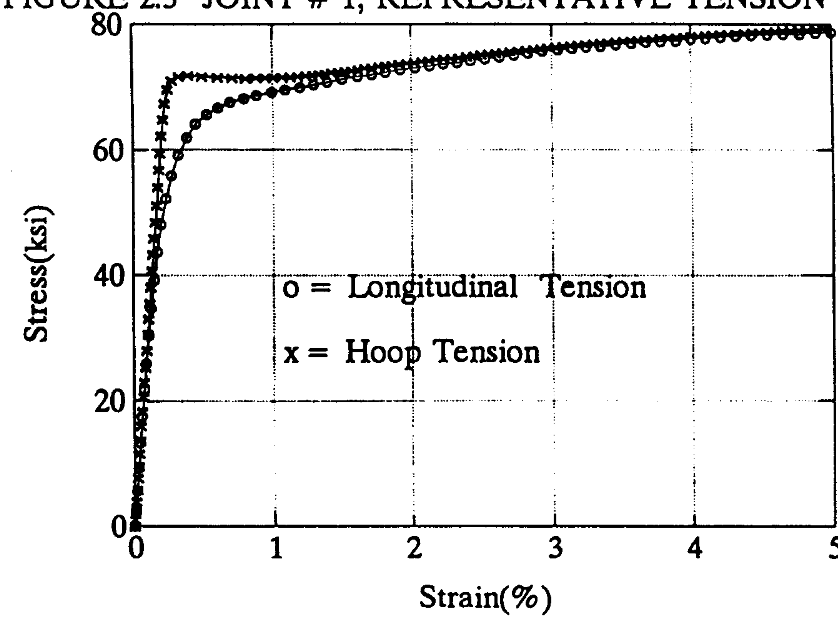


FIGURE 2.2 STRESS-STRAIN CURVE TO BE CONSIDERED IN ANALYSES

FIGURE 2.3 JOINT # 1, REPRESENTATIVE TENSION CURVES



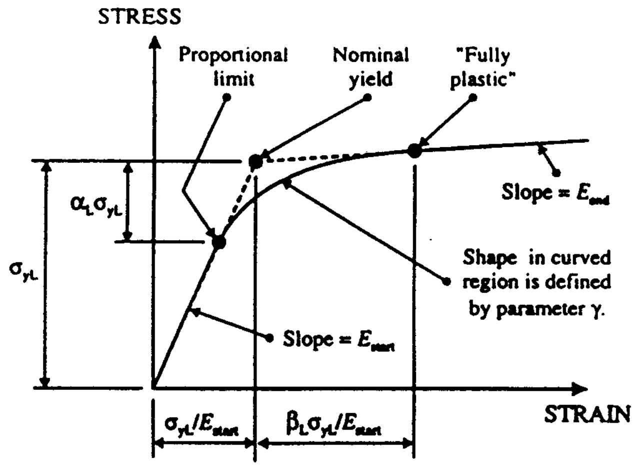


FIGURE 3.1 LONGITUDINAL STRESS-STRAIN RELATIONSHIP

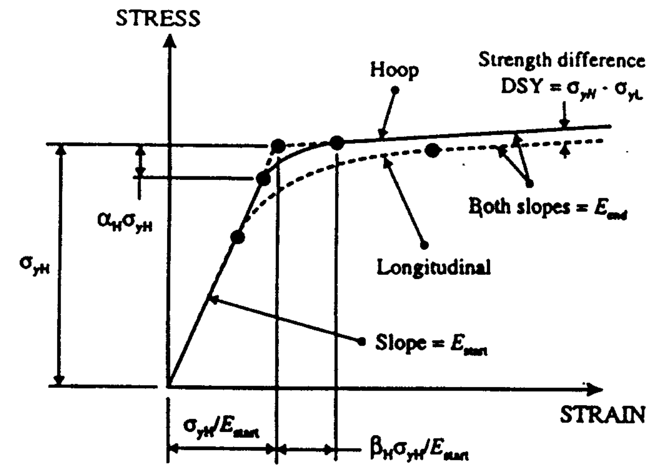


FIGURE 3.2 HOOP STRESS-STRAIN RELATIONSHIP

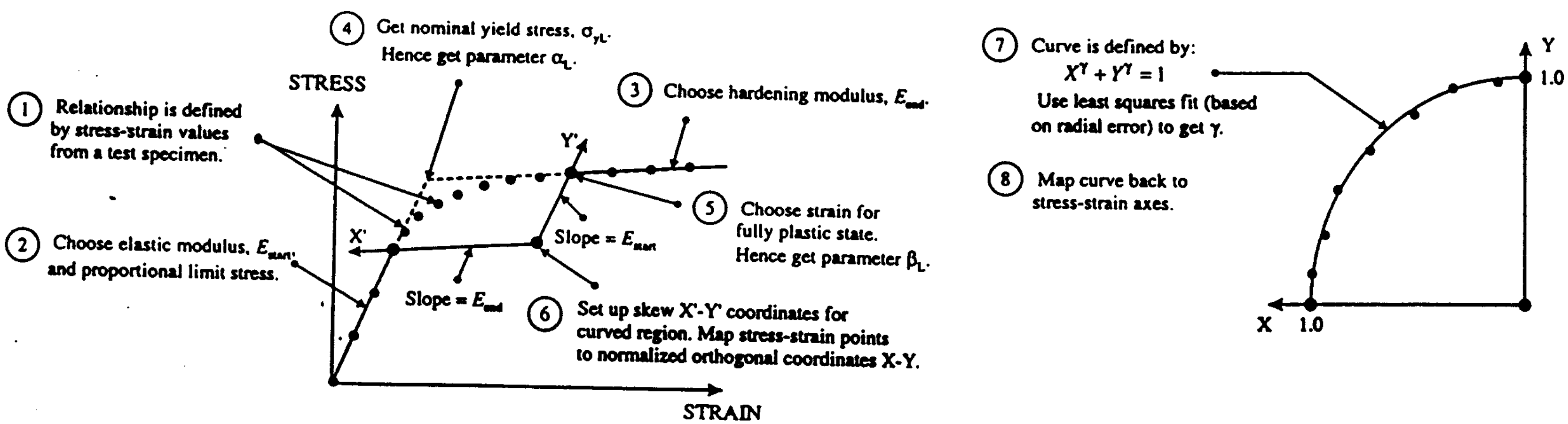


FIGURE 3.3. PROCEDURE FOR SELECTING MATERIAL PARAMETERS

Figure 3.4 Joint # 1, Analytical Tension vs Target Curves

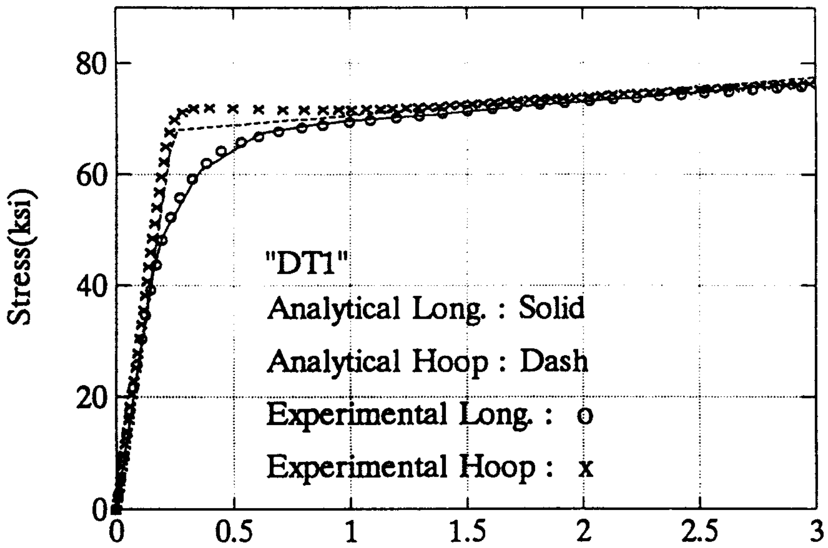


Figure 3.5 Joint # 3, Analytical Tension vs Target Curves

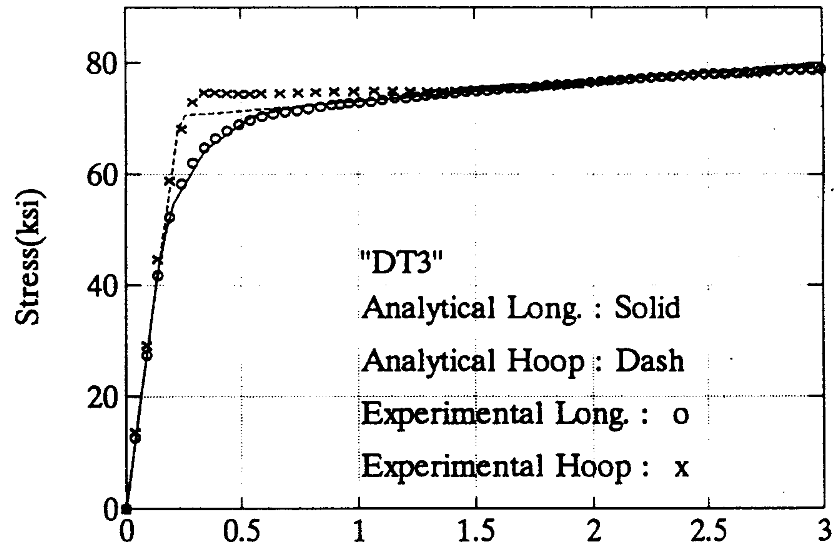


Figure 3.6 Joint # 6, Analytical Tension vs Target Curves

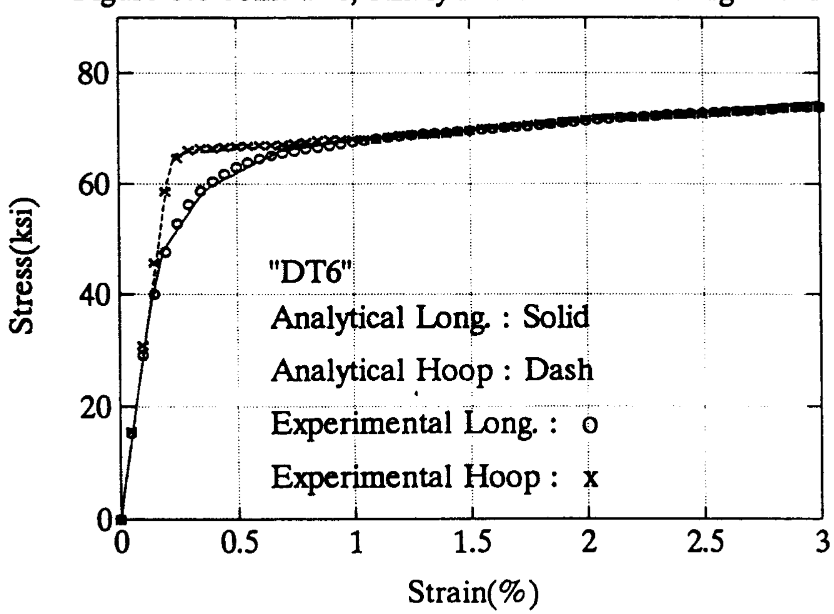


Figure 3.7 Analytical Tension vs. Digitized Joint 1007

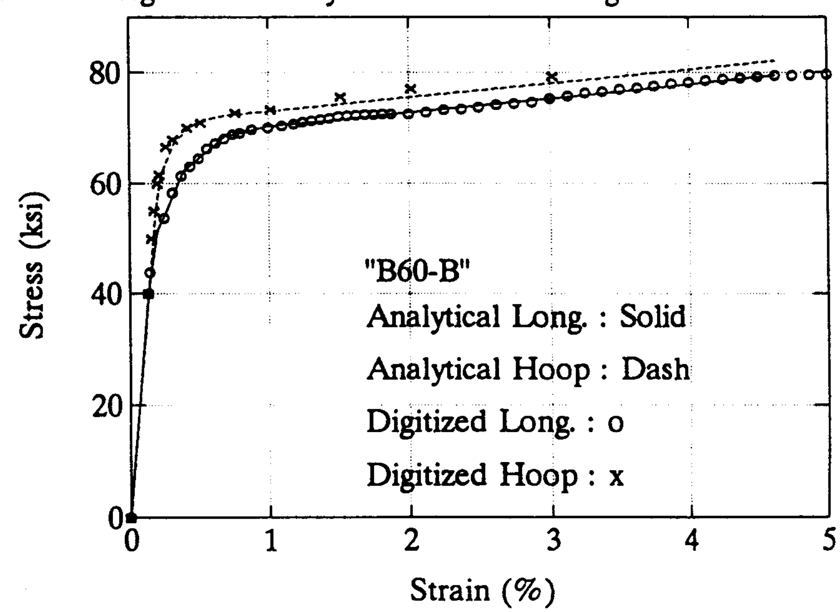


Figure 3.8 Analytical Tension vs. Digitized Joint 3743

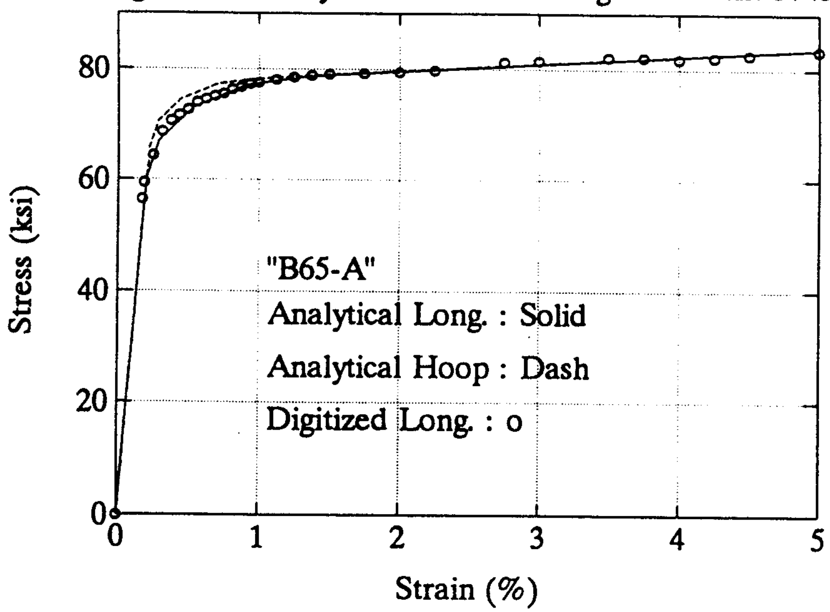
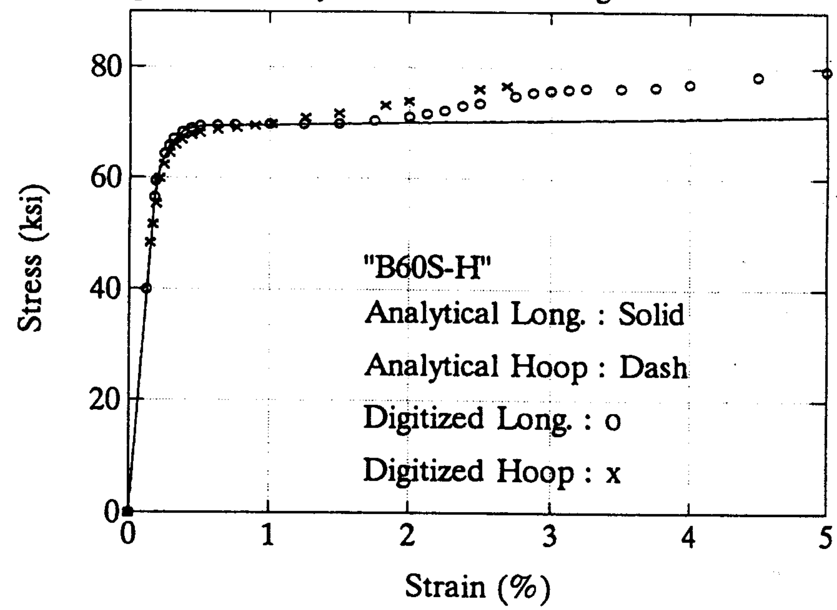
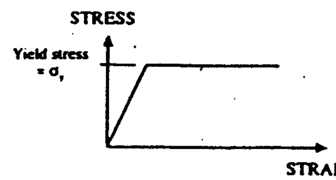
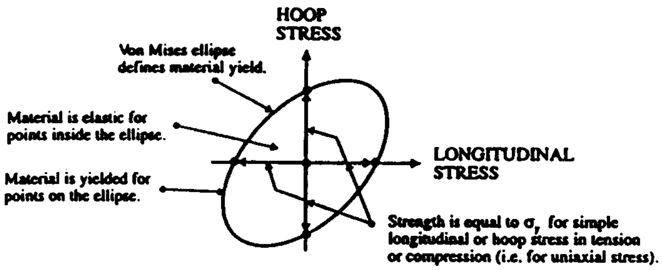


Figure 3.9 Analytical Tension vs. Digitized Joint K1212

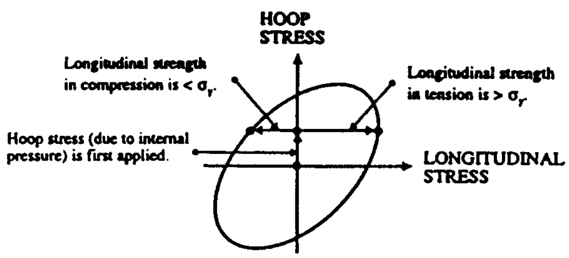




(a) Uniaxial Behavior

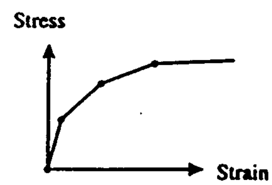


(b) Von Mises Ellipse and Yield Under Uniaxial Stress

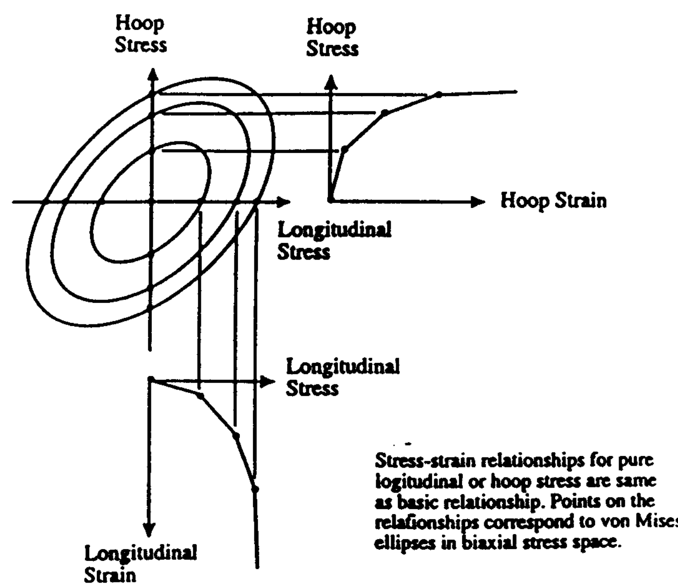


(c) Yield Under Biaxial Stress

FIGURE 4.1 EFFECT OF HOOP STRESS ON LONGITUDINAL STRENGTH



(a) Basic Uniaxial Behavior



(b) Biaxial Behavior with Uniaxial Loading

FIGURE 4.2 BASIC BIAXIAL MODEL

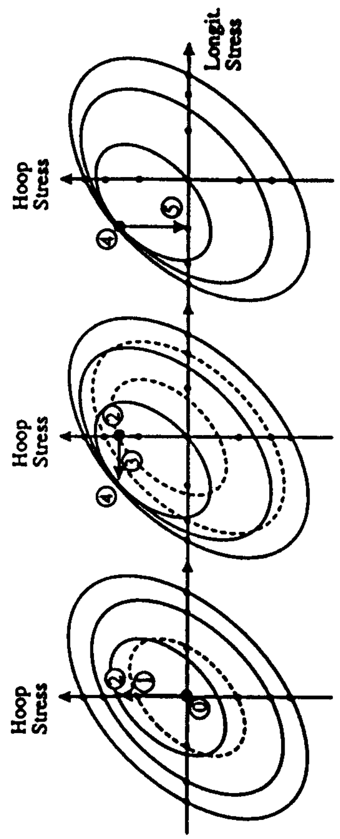
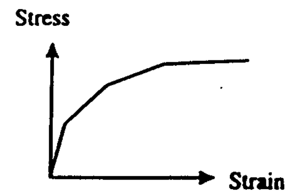
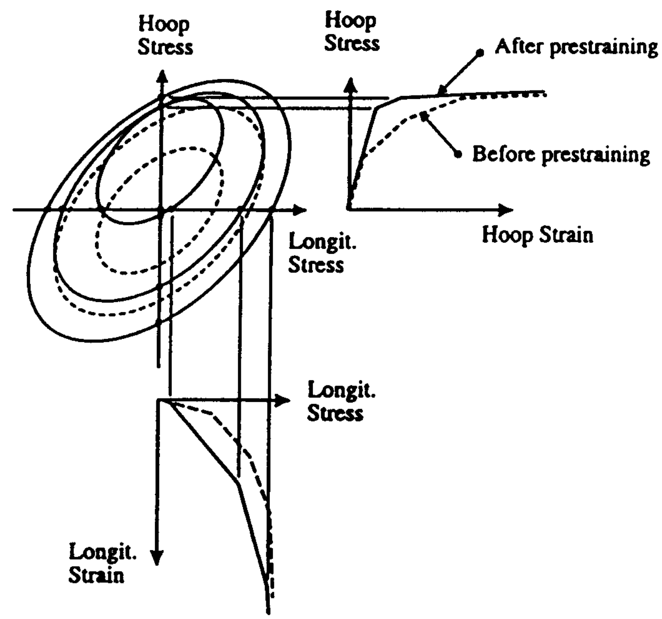


FIGURE 4.3 MOTION OF ELLIPSES IN A LOADING-UNLOADING CYCLE

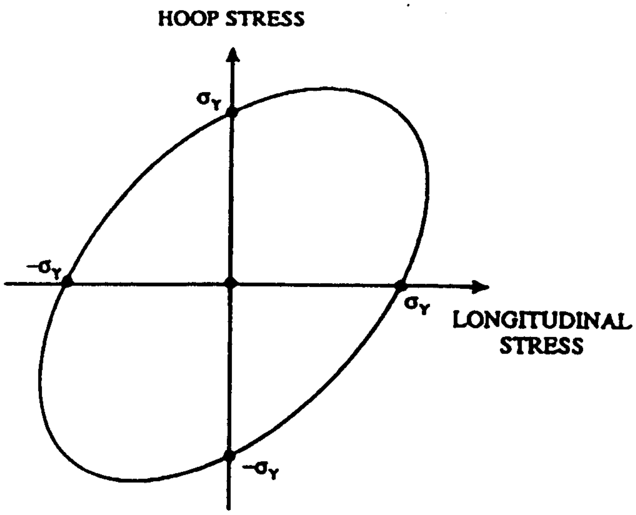


(a) Basic Uniaxial Behavior

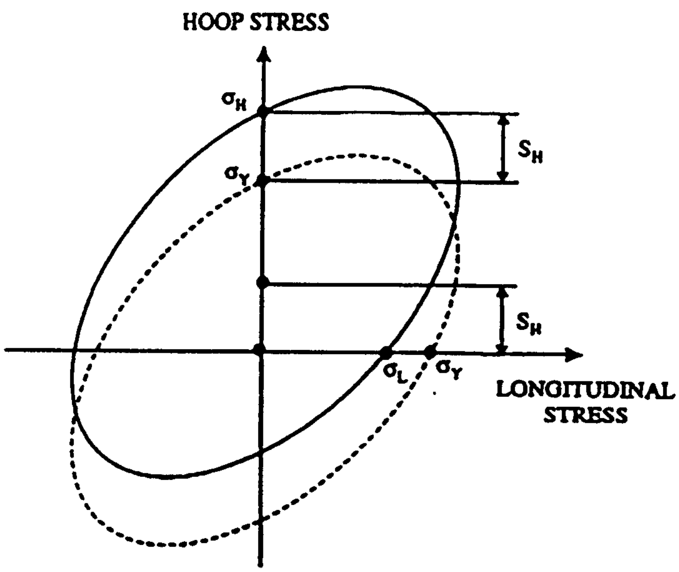


(b) Behavior After Hoop Prestraining

FIGURE 4.4 EFFECT OF HOOP PRESTRAINING ON STRESS-STRAIN RELATIONSHIPS

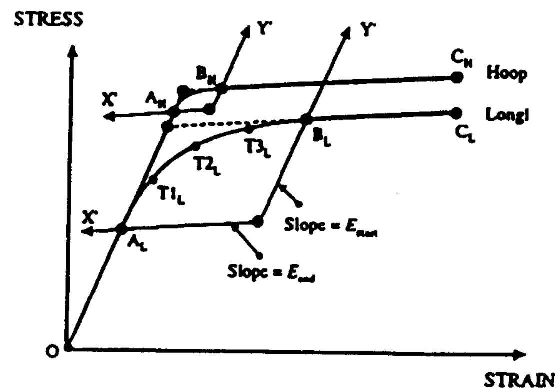


(a) Without Shift

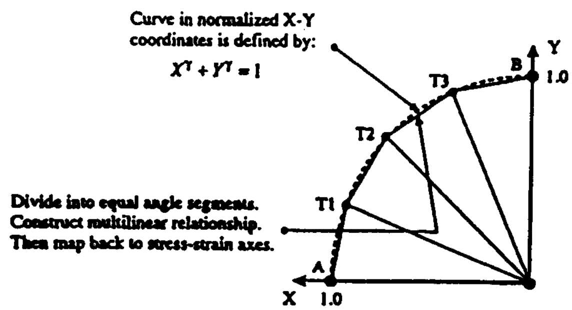


(b) With Hoop Shift

FIGURE 5.1. VON MISES ELLIPSE



(a) Points on Stress-Strain Curves



(b) Generation of Points in Transition Region

FIGURE 5.2 PROCEDURE FOR GENERATING MUTLILINEAR STRESS-STRAIN CURVES