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An Integrated Engineering Model for Prediction of Strain Demands in Pipelines Subject to Frost Heave

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ABSTRACT

Long distance pipelines are actively pursued by the industry to transport natural gas from remote arctic regions to markets. A chilled gas pipeline is one of the options to minimize the environmental impact resulting from operation of such pipelines. When a chilled gas pipeline crosses discontinuous permafrost areas, differential frost heave can occur. The result is pipe being subjected to potentially high strains, primarily in the axial direction. Reliable prediction of strain demands is one of the key components for a strain-based design process and it is essential for both ensuring pipeline integrity and facilitating life-cycle cost optimization for the design and maintenance of pipelines.

The prediction of strain demands resulting from frost heave of chilled gas pipelines involves three fundamental engineering analysis processes. They are gas hydraulic analysis, geothermal analysis and pipeline structural analysis. Not only are these three processes complex, they are also mutually interdependent. To reliably predict strain demands and fully capture the interactions among these processes, TransCanada Pipelines Ltd. (TransCanada) and its partners developed an integrated engineering model on the basis of three well established programs for the three individual engineering processes. This paper will briefly review the integrated model for strain demand prediction.

INTRODUCTION

The design, construction and operation of a buried gas pipeline in an Arctic environment pose numerous special challenges. For pipelines traversing discontinuous permafrost regions, it can be desirable to maintain the mean annual operating temperatures closely around 0 °C in order to

minimize the impact to environment and maintain pipeline integrity and safety. However, due to gas hydraulic behavior, the gas temperature will vary within a range between compressor stations. The range of operating temperature has fundamental influence on the strain demands potentially imposed on pipelines crossing discontinuous permafrost. Where a pipeline with sub-zero gas temperature crosses initially unfrozen soils, a frost heave condition may develop. Similarly, where a pipeline with above zero gas temperature crosses initially frozen soil, a thaw settlement condition may develop. In situations where differential pipe movement occurs due to either frost heave or thaw settlement, significant strains and deformations can potentially be imposed on the buried pipeline. Hence, the problems associated with frost heave and/or thaw settlement are a major concern for the design of an Arctic gas pipeline.

Recognizing the challenges related to frost heave and thaw settlement, an alternative design methodology is required to design and construct pipelines in permafrost areas. Because frost heave and thaw settlement are displacement-controlled loads, strain-based design (Zhou et. al., 2006) lends itself as one of the appropriate alternative design methodologies to established supplemental design criteria in addition to the conventional design criteria for pressure containment. The principle of strain-based design is to ensure strain demands do not exceed strain capacity with an adequate safety margin over the entire operating life of the pipelines. Applications of strain-based design to chilled gas pipelines rely on the capability to predict strain demands and strain capacities with validated engineering models. The following sections describe an integrated strain demand model specifically developed for frost heave.

OVERVIEW OF STRAIN DEMAND PREDICTION FOR FROST HEAVE

In general, three steps are required for strain demand prediction and they are: a) hazard assessment to quantify the mechanism and magnitude of the hazard; b) load transfer process to quantify the mechanism and magnitude of load transferred to the pipeline; and c) pipeline structural analysis model to quantify the strain demand. In the case of differential frost heave, strain demands are dependent on many factors, including soil type and mechanical properties; soil temperature and thermal properties; gas operating temperature; climate conditions (ambient temperature and snow depth); pipe specification; and pipe material properties. The overall process for determination of strain demands is an integration of three key modeling processes and they are: a) gas hydraulic simulation; b) geothermal analysis; and c) pipeline structural analysis.

Hydraulic simulation of gas pipelines is well developed based on three basic principles: a) conservation of mass (continuity); b) conservation of momentum (Newton's second law); and c) conservation of energy (second law of thermodynamics). Various models and equations of state (EOS) are available. For temperature sensitive applications such as hydraulic simulation of chilled gas pipelines, it is critical to consider energy flow both into and out of the system, which is dependent on the temperature differential between the gas and surrounding ground temperatures, and the energy accumulation within the system. For the specific purpose of supporting the transportation of Arctic gas, TransCanada (Foothills Pipelines) has developed a hydraulic simulator, TempFlo, which has been used for various system design and regulatory applications and validated during operation against monitoring data in the last 25 years. TempFlo is fully developed hydraulic analysis program capable of simulating a pipeline system including all surface facilities such as compressor stations, chillers and heaters. It has a built-in database for common equipment.

To reliably account for the heat transfer between the pipeline and surrounding soil, the thermal state of the soil needs to be modeled via geothermal analysis. TransCanada uses under license a state-of-art geothermal analysis program, TQuest (Northern Engineering & Scientific ©1989-2005). In a typical 2-dimensional geothermal analysis, the soil is represented by soil slices along the pipeline as illustrated in Figure 1 where heat transfer, temperature distribution, surface and climate condition, soil thermal properties and phase change are all addressed. With empirical correlations between the frost heave and thermal characteristics and other factors (Konrad and Morgenstern, 1980), the geothermal analysis programs, such as TQuest, are also able to predict the frost heave accumulation over the entire operating life of the pipeline.

With the capability to quantify frost heave, the strain demands can be predicted via pipe-soil interaction analysis. In a typical pipe-soil interaction analysis, the pipeline is represented by a series of pipe elements, and soil is represented by a series of soil spring elements. There are two

components that are critical for reliable strain demand prediction. One is the definition of soil springs, and in the case of frost heave, the uplift spring, which is often determined via a combination of laboratory tests, field tests and specialized numerical simulations (Nixon, J.F., 1998; Liu et al. 2004). The other is the pipeline structural analysis for which TransCanada has licensed a well established program, PIPLIN (SSD, Inc., 2002).

While the three independent analysis processes conceptually represent the entire process for strain demand prediction, they fail to fully capture the interdependencies, including:

- Gas temperature and soil temperature are mutually dependent to each other. However, gas temperature is determined in hydraulic analysis and soil temperature is determined in geothermal analysis.
- It is well established that the rate and accumulated magnitude of frost heave are dependent on the magnitude of pressure at frost front. A part of the pressure at the frost front results from pipeline deformation which is in turn induced by frost heave. However, frost heave is determined in geothermal analysis and pipeline deformation is determined in pipeline structural analysis.
- It has been demonstrated that pipeline deformation is significantly influenced by soil uplift resistance which is in turn influenced by soil temperature. Similarly, soil temperature is determined in geothermal analysis while pipeline deformation and uplift resistance are determined in pipe-soil interaction analysis.

To fully capture the interdependencies and simplify data transfer among the analysis processes, TransCanada and its partners have developed integrated models for strain demand prediction: GeoFlow and GeoPipe. GeoFlow is a fully integrated model for hydraulic analysis and geothermal analysis based on programs TempFlo and TQuest. Similarly, GeoPipe is a fully integrated model for geothermal analysis and pipeline structural analysis based on programs TQuest and PIPLIN. GeoFlow and GeoPipe are completely compatible and consistent with each other in terms of data format, input, output, and definition of soil and pipe. They share the common data processing and display interface. GeoFlow and GeoPipe are, however, often used on quite different scales. GeoFlow's prime purpose is to provide hydraulic conditions along the entire length of a pipeline system, which could be thousands of kilometers, for the operating conditions expected over the entire life of the pipeline. This could include multiple station facilities with multiple expansion phases. GeoPipe is often focused on a relatively short segment of a pipeline system in order to predict frost heave and strain demands more reliably and accurately.

INTEGRATED HYDRAULIC SIMULATION AND GEOTHERMAL ANALYSIS – GEOFLOW

As was explained in the previous section, GeoFlow is the product of coupling a geothermal model and a hydraulic model. The GeoFlow model includes the finite-element geothermal model TQuest and the finite-difference hydraulic

model TempFlo. A significant advantage that GeoFlow has is that it incorporates fully functional mature simulation models, both of which were developed for northern projects and refined over time.

The temperature of the gas as it flows through the pipeline is a function of both the Joules-Thomson effect and the pipe to soil heat transfer, or heat flux. The Joules-Thomson effect is defined as the net cooling that results from expansion of the gas as it moves down the pipeline and is accurately predicted by the hydraulic model. Heat flux is the transfer of energy from the pipe to the surrounding soil and is dependent upon both the flowing gas temperature and the ground temperature surrounding the pipe. Heat flux is determined by the geothermal model using an iterative calculation method, ensuring that an overall energy balance is maintained between time steps. The calculated heat flux is passed back to the hydraulic model for use in its next iteration.

Figure 1 shows how GeoFlow idealizes coupled simulations. GeoFlow assumes the pipeline and alignment can be represented by discrete “slices” or volumes of soil. A hydraulic node is defined at the beginning and end of each of these slices. It is the hydraulic model that connects the slices together. From a geothermal perspective, adjacent slices are only connected by the hydraulic nodes. In other words, all heat exchange in the pipeline direction occur inside of the pipeline via gas hydraulic process and there is no heat exchange in the pipeline direction either within a soil slice or between adjacent soil slices.

It is recognized that the series of 2-dimensional soil slices do not simulate the heat transfer from a soil slice to its adjacent soil slices. However, the heat transfer between soil slices is limited to the local transition zone between soil slices. As a result, this limitation has limited influence on gas hydraulics and soil thermal states at the pipeline system level. However, it could potentially have measurable influence on frost heave prediction and pipeline structural analysis which are focused on a relatively short segment. The effects of heat transfer between soil slices can be evaluated by incorporating a full 3-dimensional geothermal model.

The length of pipeline represented by each slice is defined by both geotechnical and hydraulic considerations. Typically, slice lengths are first defined to reasonably represent field conditions as defined by pipeline alignment sheets, such as frozen/unfrozen thermal states and boundaries, soil stratigraphy, climate, pipeline burial configuration, etc. Slices are also inserted where a change in grid type is needed. No two slices need be alike in soil layering and properties, initial temperatures, alignment geometry, climate application, etc. An example would be a change from a moderately coarse grid, i.e., suitable for simulating frozen soil, to a fine grid for simulating unfrozen, potentially frost-heaving soil for a chilled gas pipeline. Next, slices are often subdivided to place hydraulic nodes at key locations such as at above-ground facilities or to properly specify the hydraulic profile to accommodate elevation changes along the pipeline. Because of the architecture of GeoFlow, adding greater resolution to the simulation, i.e., by using either finer geothermal grids or more slices, is relatively easy.

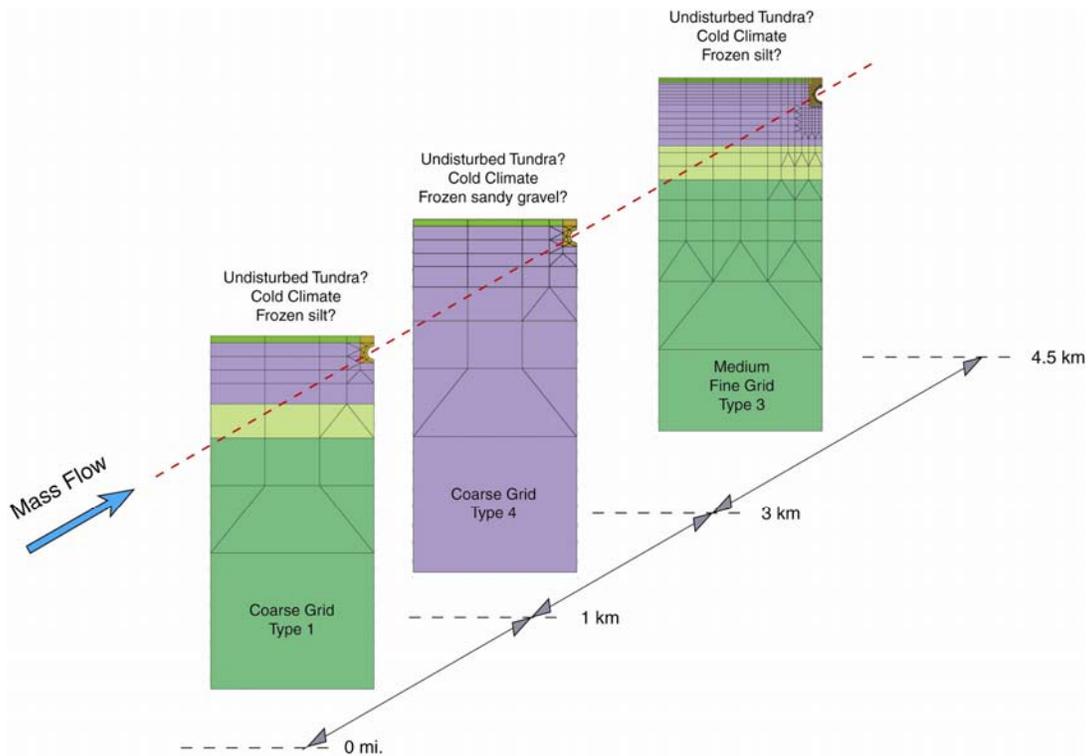


Figure 1 GeoFlow's Slice Representation of the Pipeline Alignment

During the GeoFlow simulation, transient heat fluxes into or out of the pipeline for each slice are calculated by the finite-element thermal model; once fluxes for all slices are ready they are passed (through a common computer memory area) to the hydraulic model; the hydraulic model uses the fluxes to calculate gas pressures and temperatures and passes these data back to the thermal model, again through the common memory area. The process marches forward through time: the thermal model assuming that for each slice the gas temperature remains constant over the time-step; the hydraulic model assuming the heat flux remains constant over the time-step. The size of time-steps can be defined in accordance of expected variation and accuracy. However time-steps on the order of 73 hours each (120 time-steps per year) are commonly used.

GeoFlow has the capability to model an entire pipeline system, including compression and other above ground facilities and execute in a single simulation. One of the significant benefits of GeoFlow is that it can help define the effects of variations in pipeline operation temperature on pipeline frost heave and thaw settlement; it can show how changes in pipeline operation temperature propagate along the system; it can show how changes in system capacity may affect frost heave, thaw settlement and terrain stability. GeoFlow can also help to define the importance of gathering and processing "better" field/input data and the effect of reasonable statistical variations in input data.

Another important feature of GeoFlow is the ability to modify the pipeline operating conditions at specified times throughout the simulation. This capability allows for determining the effects of adjusted flows or facility changes, such as future expansions. In addition, the effect that outages of compression or other facilities will have on the soil temperature profiles can also be predicted. With this information, the pipeline can be designed to withstand the effects of soils related external forces such as frost heave, either through revised operating strategies or by incorporating special design measures in the affected area. GeoFlow can also be used in the operations phase of the project to ensure operational decisions are made with a better understanding of the impacts they may have on the pipeline and soils.

All of the capabilities built into the finite-element thermal model, such as conservation of energy, adaptive timestep, frost heave, thaw settlement, heatpipes, multiple pipes, 2D and 3D, etc., are available for a GeoFlow simulation.

Similarly, all of the capabilities built into the hydraulic model are also fully available for a GeoFlow simulation. To provide better gas temperature prediction, which is an important consideration in the modeling of northern pipelines, TempFlo incorporates the conservation of energy equation as part of its node to node calculations. Other advanced features which can be used while performing GeoFlow simulations include:

1. the AGA-8 Equation of State as an option to the BWRS Equation of State in order to obtain more accurate density calculations at higher pressures,
2. a compressor/power turbine speed matching routine from which wheel efficiencies, powers, and fuel requirements are automatically calculated through the use of equations tuned to the equipment being used,
3. after compression cooling of the gas which can be specified using either aerial coolers or a propane refrigeration chilling cycle, and
4. pressure regulator and heating station routines

GeoFlow results for any selected pipeline segment are passed to GeoPipe for prediction of differential frost heave and resultant strain demands.

INTEGRATED FROST HEAVE PREDICTION AND PIPELINE STRUCTURAL ANALYSIS – GEOPIPE

As stated in the previous sections, strain demand prediction for frost heave involves three analysis processes: gas hydraulic simulation, geothermal analysis and pipeline structural analysis. Until recently, most analysis work in this area involved "uncoupled" analysis of hydraulics, geothermal and pipe deformation issues.

In an uncoupled, geothermal/structural analysis, geotechnical input required for a pipe-soil deformation analysis is typically established from an independent geothermal analysis model which is essentially a finite element analysis of a given section of the chilled pipeline based on the pipe temperature and soil parameters such as grain size, moisture content, location of the water table, etc. The geothermal heat transfer analysis is used to predict how the frost bulb grows around a chilled buried pipe with time and to estimate resulting heave. These analyses are typically carried out as step-by-step time history analyses assuming an unrestrained, "free-heave" or constant pressure condition (e.g., pressure feedback effects from the pipe are neglected). By processing the geothermal simulation results over the mesh at each time step (e.g., tracking the location of the 0 °C temperature isotherm), it is possible to develop measures of the frost bulb geometry (i.e., depth and width) at the end of each analysis time step.

Using the results (in the form of curve fits of bulb depth, width and heave) from a geothermal simulation, the pipe-soil deformation analyses typically involves a multi-year, step-by-step time history simulation of differential frost heave with or without consideration of uplift and/or creep effects. The deformation analysis is carried out independently from the geothermal simulation with the effects of pressure feedback approximated indirectly as a pressure dependent term in the incremental heave formulation. The results from deformation analyses can be used to estimate the time and/or heave required for the pipe strain demands to reach specified measures of the pipe strain capacity. Sensitivity studies on frost heave span and transition, length, the soil uplift resistance and other effect are typically carried out to study the pipeline design.

As a part of the integrated model, TransCanada developed GeoPipe by integrating geothermal analysis program TQuest and pipeline deformation analysis program PIPLIN. In GeoPipe, the basic coupling involves running a single analysis time step in the geothermal model, passing via shared-computer memory the results to the pipe structural model, running the same time step in the pipe structural model and then passing the results (i.e., the pipe deformation and pressure feedback) back to the geothermal model. The process is then repeated for subsequent time steps and the geothermal and deformation analyses are advanced through time until the analysis duration is complete.

The main benefits of the geothermal and pipe structural model integration in GeoPipe include:

1. Real time temperature profile for uplift soil springs. In a typical (uncoupled) PIPLIN frost heave analysis which uses temperature dependent uplift soil spring properties, it is a usual practice to calculate a maximum (winter) uplift strength and a minimum (summer) uplift strength and to assume a linear variation of uplift strength between these summer and winter strengths (i.e., during the spring and fall seasons). In this fashion, the uplift strength is controlled using an “uplift soil temperature” which varies as a pseudo-sign wave over the course of a typical annual cycle (i.e., constant strength values during the summers and winters with a piecewise linear variation between these extremes during the springs and falls). Based on the temperature isotherms (contours) around the pipe from the geothermal analysis model, it is possible to develop profiles of the temperature of the soil above the pipe or the temperature extending horizontally from the pipe centerline during each time step of the analysis or, in a less general sense, for a typical summer and winter season. Given these soil temperature profiles and a relationship between soil uplift strength and different measures of the soil temperature profile, it would be possible to develop estimates of the corresponding summer and winter seasonal strength variations for the uplift spring. This would eliminate the need for an idealized “pseudo-sine wave” shape over a “typical” annual cycle and could provide more accurate estimates of the seasonal variation of uplift strength. A connection between the TQuest soil temperatures around the pipe and the uplift soil temperature used by PIPLIN also provides a framework for simulation of non-typical annual temperature cycles and pipe temperature cycling effects.
2. Direct feedback due to pressures at the frost front. Most geothermal analyses assume free-heave or constant pressure conditions. The rate of heave is known to depend on the pressure at the frost front with less heave occurring for greater pressures and more heave occurring for lower pressures. In order to approximate this effect using the uncoupled simulation approach, frost heave time histories from geothermal simulations at different pressures are curve fit using a single function that depends on time and pressure. Using the curve fit, PIPLIN accounts for the pressure sensitivity at each spring in each step using the

computed pressures in a pressure term used to scale the heave increment. Because estimates of the pressure at the base of the frost bulb can be directly computed at each step of the PIPLIN analysis using the bulb depth provided by TQuest, it is possible to provide this pressure result as direct feedback to the geothermal model on a step-by-step basis in a coupled modeling approach. This would avoid the need for multiple geothermal simulations at different pressures, eliminate any approximations due to curve fitting, and provide more accurate estimates of the heave as a function of time.

3. Rational evaluation of thermal cycling. Thermal cycling of a chilled buried pipe has been discussed as a potential way to mitigate the effects of frost heave (i.e., periodically running the pipe in a warm mode to degrade the frost bulb). Thermal cycling of this sort can be directly simulated in the geothermal model, again on a step-by-step time history basis. The coupled geothermal-deformation model would provide an excellent basis for evaluating the effects of thermal cycling (i.e., the “degraded” bulb geometry would be input directly to the deformation model at each time step). In fact, a coupled analysis approach is probably the only way that the effects of thermal cycling could be rationally considered.
4. Ability to easily model a multi-year ramp-up of system capacity, seasonal demand and staged expansions.
5. Less labor intensive analyses with fewer steps requiring manual intervention.
6. Potentially easy integration with real-time, optimized system operation.

GeoFlow and GeoPipe include two frost-heave formulations: the NWA Heave Correlation Equation (ANNGTC, 1984) and the Ice Segregation Potential equation (Konrad and Morgenstern, 1980). In the NWA equation, heave rate (dh/dt) is a function of soil constituents, pressure at the frost front and frost-penetration rate (dx/dt). In the SP equation, heave rate (dh/dt) is a function of the thermal gradient (dT/dx) and the soil’s SP, which is a function of the stress at the frost front. Stress at the frost front is a function of pipe burial depth, frost bulb depth, frost bulb width, groundwater table elevation, soil layering and pipe loading from PIPLIN. TQuest keeps track of heave as a function of frost-bulb depth. If the frost-bulb retreats in a given time step (due to pipe operating temperature, warming in the domain, etc.), TQuest walks back down the heave vs. frost-bulb depth curve, reporting the heave that corresponds to the current frost-bulb depth.

TYPICAL RESULTS

The volume of data that naturally results from GeoFlow and GeoPipe simulations is typically large as all hydraulic, geothermal and structural data are collected at each time step. To facilitate the post processing of this simulation data, a Graphic User Interface (GUI) was developed. This GUI, which is also used to manage the user’s execution in the

running of GeoFlow, effectively organizes, processes, and displays input and output data. With the functionality that the GUI provides, the user has the capability of displaying data as a function of time, distance, or cross-sectional area.

The graphical displays that are generated greatly assist the user in the interpretation of the data. An example case has been created to demonstrate both the typical results that GeoFlow generates and the use of its graphical capabilities. The example case assumes a pipeline approximately 100 km in length that traverses through a region of discontinuous permafrost comprising 75 changes to geotechnical conditions. When coupled with elevation points, this resulted in a file containing 211 slices. The inlet conditions were adjusted monthly with the temperature and pressure reduced in the winter, but with the flow increased. These conditions were ramped for each 3 day time step, and were repeated on an annual cycle over a 5 year period. The length of simulation can be specified by the user, but good repeatability of temperature results is generally observed after the third year provided simulation conditions are unchanged.

Using year 5 data, the following figures show the temperature and pressure profiles occurring at the times of temperature extremes, mid February and mid August. Also shown on the temperature profile is the predicted annual average temperature.

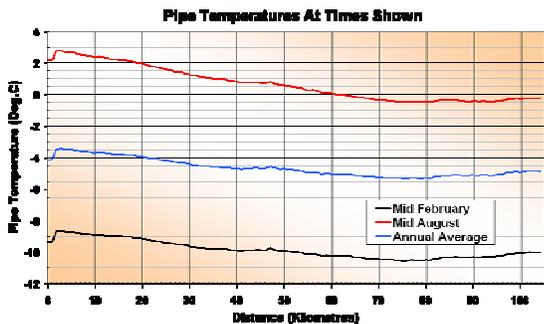


Figure 2 Illustration of Temperature Profile along a Pipeline

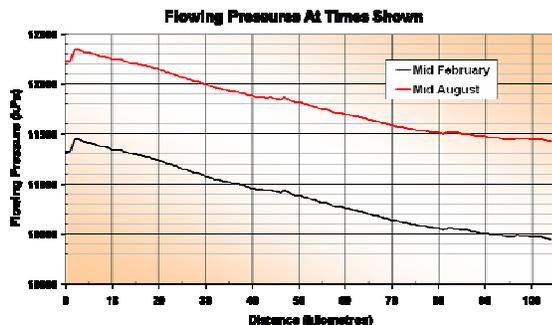


Figure 3 Illustration of Pressure Profile along a Pipeline

The information can also be displayed as a function of time for any node location. In this example, the location

exposed to the lowest pipe temperatures was at kilometer post 75.8. The pipe pressure and temperature at this location for the first five years of service are shown in the following plots.

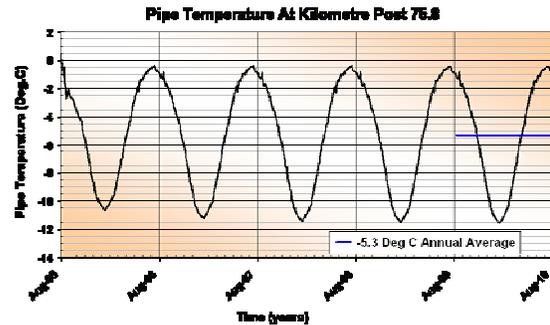


Figure 4 Illustration of Temperature History for a Specified Location

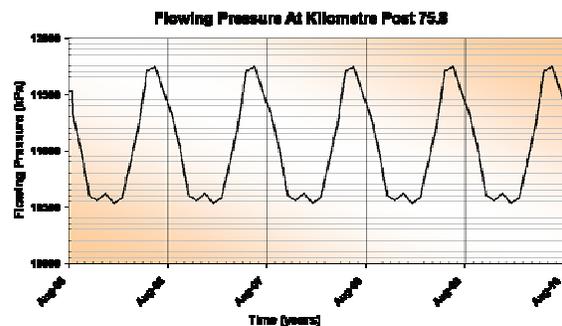


Figure 5 Illustration of Pressure History for a Specified Location

As this location isn't in a permafrost region, there is a greater than average potential for high frost heave. The pressure and temperature data collected by GeoFlow for this or any other location can then be transferred to GeoPipe for further assessment. While GeoFlow has the capability of predicting the degree of frost bulb growth, this function is preferably performed by GeoPipe, which is better suited for longer term simulations, typically 35 years, where pipe stress and strain calculations are performed simultaneously.

Another important feature is the capability of producing isotherms, both profiles and cross-sections to visually interpret the results. The user, reviewing a chronological series of isotherms can observe the changes occurring in the soils surrounding the pipe throughout any period of time. To illustrate this feature, two instances for each of the typical isothermal profiles and the typical cross-sectional isotherms have been provided. The extreme temperature conditions in the example case were selected, with kilometer post 75.8 again being used for the cross-sectional isotherms.

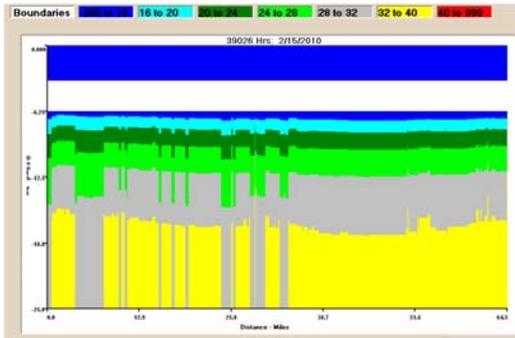


Figure 6 Illustration of Isothermal Profile in Mid-February

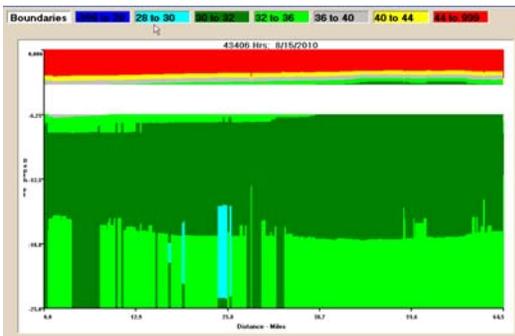


Figure 7 Illustration of Isothermal Profile in Mid-August

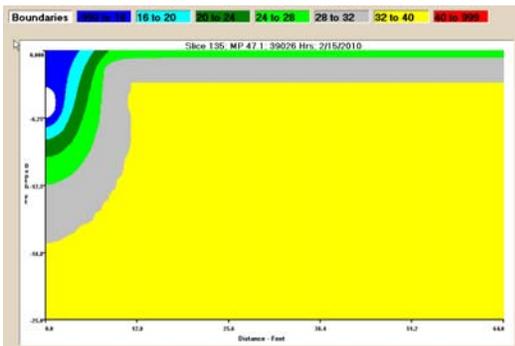


Figure 8 Illustration of Cross-Sectional Isothermal in Mid-February for a Specified Location

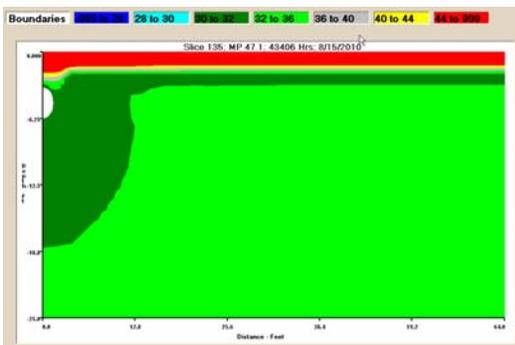


Figure 9 Illustration of Cross-Sectional Isothermal in Mid-August for a Specified Location

Note that the colors and color ranges can be changed to suit either the user's preference or to show the level of definition required.

With the gas hydraulics defined at the pipeline system level through GeoFlow simulations, representative segments of the pipeline are selected for further refined frost heave simulations and pipeline structural analysis through GeoPipe. In addition to more refined isothermal results similar to those illustrated in Figures 6 to 9, typical GeoPipe results would include predicted frost heave accumulation, frost bulb width and depth, and ultimately the strain demands over the specified operating life as illustrated in Figures 10 to 12.

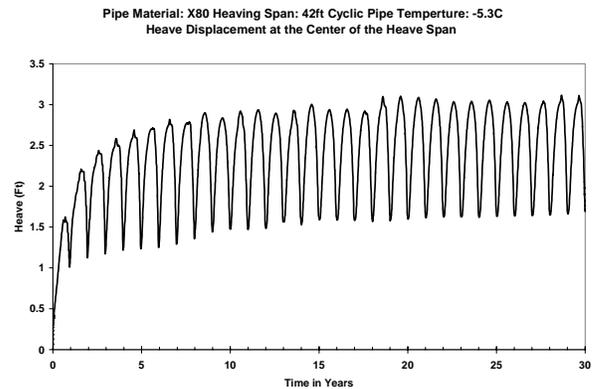


Figure 10 Illustration of Predicted Frost Heave Accumulation over 30 Years Operating Life

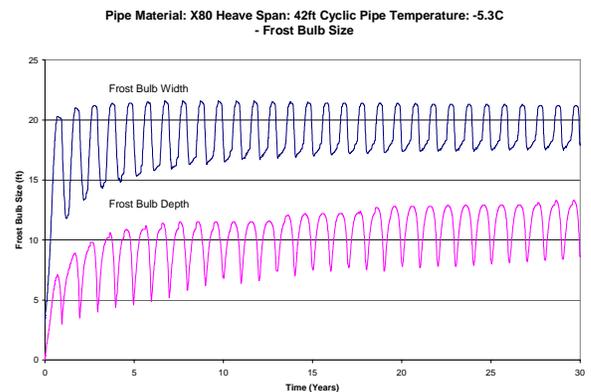


Figure 11 Illustrations of Predicted Frost Bulb Width and Depth over 30 Years Operating Life

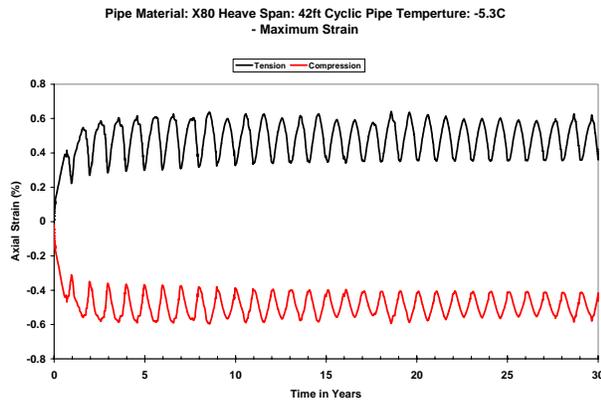


Figure 12 Illustrations of Predicted Strain Demands over 30Years Operating Life

While this example case has shown the basic typical results for a section of pipe, the power of GeoFlow and GeoPipe as an integrated model, as previously stated, is in its capability to perform these analyses for a complete pipeline system with facility or operational changes incorporated at specified times throughout. The integrated model for prediction of strain demands induced by frost heave represents a complete and powerful design and operational tool. When it is used appropriately within a strain-based design framework it can adequately ensure structural integrity of the pipeline against frost heave over the entire operating life.

CONCLUSIONS

This paper presented an integrated model for strain demand prediction. The integrated model was developed by TransCanada and its partners based on three well established programs: TempFlo for gas hydraulic simulation, TQuest for geothermal analysis and frost heave prediction, and PIPLIN for pipe-soil interaction analysis. The integrated model has the capability to simulate a complete pipeline system, including all the surface facilities and various operation and expansion options. It is able to fully capture the mutual dependencies among the gas hydraulics, geothermal state, frost heave growth and accumulation, pipeline deformation and strain demand. The integrated model has been used by TransCanada for planning and design of pipelines in discontinuous permafrost areas.

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